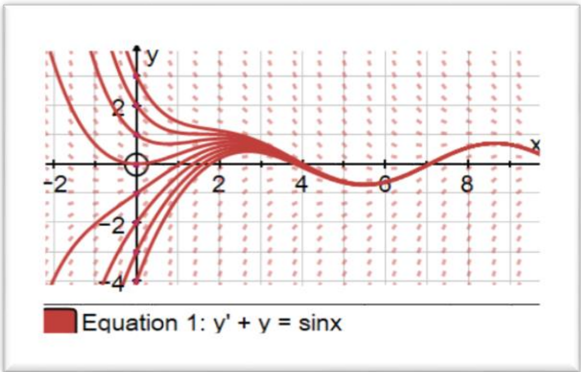
2^{nd} Order Differential Equations and SHM

A review of the 1^{st} order linear D.E.

On a 2D page in Autograph, with slow plot on:

Enter		*Solution*		
$y' = 0$	=>	**$y = c$**		
$y' = 1$	=>	**$y = x + c$**		
$y' = x$	=>	**$y = \frac{1}{2}x^2 + c$**		
$y' = -x$	=>	**$y = -\frac{1}{2}x^2 + c$**		
$y' = y$	=>	**$y = ae^x$**		
$y' = -y$	=>	**$y = ae^{-x}$**	=>	**$y' + y = 0$**

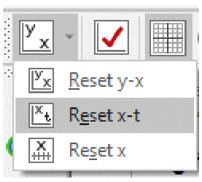
In this implicit linear form, $y' + ky = f(x)$
$y' + ky$ is the 'complimentary function' (C.F.) and
$f(x)$ is the 'particular integral' (P.I.)

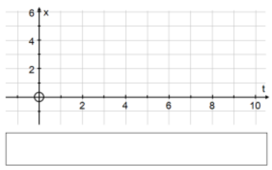


This shows nicely that the C.F. is an exponential decay, down to the steady state (P.I.).

The 2^{nd} order linear D.E

Most 2^{nd} order D.E.s refer to variables against time. You can reset the variables to 'x' against 't', and then entering **x'** or **x"** shows up as **$\dot{x}$** and **$\ddot{x}$**.





On a 2D page in Autograph, with slow plot on:

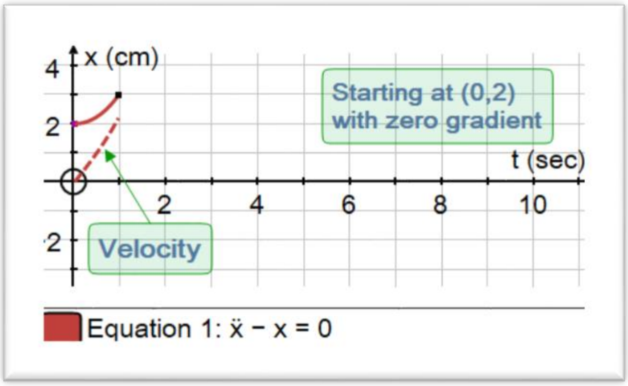
Enter				*Solution*
$\ddot{x} = 0$	=>	**$\dot{x} = a$**	=>	**$x = at + b$**
				Family of straight lines
$\ddot{x} = 1$	=>	**$\dot{x} = t + a$**	=>	**$x = \frac{1}{2}t^2 + at + b$**
				Family of parabolas
$\ddot{x} = x$	=>	**$\ddot{x} - x = 0$**	=>	**$x = ae^t + be^{-t}$**
				General exponential
$\ddot{x} = -x$	=>	**$\ddot{x} + x = 0$**	=>	**$x = a\sin t + b\cos t$**
				Simple Harmonic Motion

Start-up options for 2^{nd} order D.E.s

Manual: click anywhere to start a solution
Point: use a selected (moveable) point as starter
Point set: create a set of start points, eg along y-axis
Also: the option to draw the gradient (velocity).

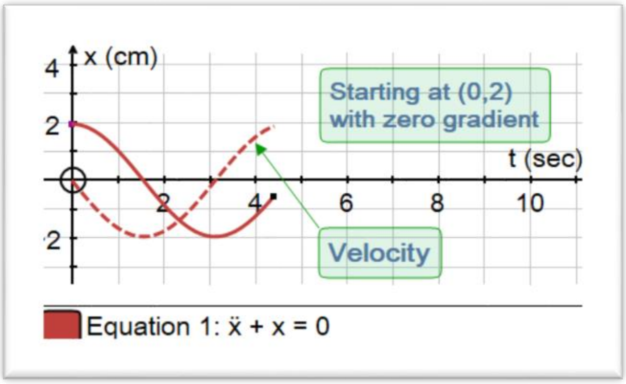
The link with SHM

$\ddot{x} - x = 0$ This can be interpreted as follows: A body's acceleration is equal to its distance from the 't' axis. Hence as 't' increases, so the acceleration away from the 't' axis increases.



$\ddot{x} + x = 0$ Here, a body's acceleration is equal to MINUS its distance from the 't' axis. As 't' increases, so the acceleration decreases to zero on the 't' axis, when its velocity is maximum.

As soon as it crosses the 't' axis the acceleration switches and it slows down, giving rise to Simple Harmonic Motion.



Note: **$a\sin t + b\cos t$** => **$A\sin(t + \phi)$**
where **$a = A\cos t$, $b = A\sin t$** => **$\tan\phi = b/a$**

More generally **$\ddot{x} + \omega^2 x = 0$ => $x = A\sin(\omega t + \phi)$**
ω = angular frequency (rad/sec) = $2\pi f$
f = frequency (Hz) T = period (sec) = $1/f$
A = amplitude (cm) ϕ = phase (rad)

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