



Autograph

version 3

and AP CALCULUS

Autograph is spectacular dynamic software from the UK that allows teachers to visualise many of the mathematical topics that occur in the AP CALCULUS AB and CALCULUS BC courses.

1st and 2nd DERIVATIVE

Equation 1: $y = x^3 - 3x - 1$
 Gradient 1: 1st Gradient of $y = x^3 - 3x - 1$
 Gradient 2: 2nd Gradient of $y = x^3 - 3x - 1$

MAX and MIN

Tin Can, Volume = 3
 Minimum $S = 277.5 \text{ cm}^2$ at $r = 3.837 \text{ cm}$

Equation 1: $s = 2\pi r^2 + 710r$
 Equation 2: $s = 2\pi r^2$
 Equation 3: $s = 710r$

EXPONENTIAL FUNCTION

1st Gradient of $y = a^x$
 Integral Function of $y = a^x$

Constant Controller: [Page - 1]
 Options: a = 2, Step: 0.1

$y = a^x$
 Integral Function of $y = a^x$
 1st Gradient of $y = a^x$

PARAMETRIC Coordinates

A: $(\phi - \sin\phi, 1 - \sin\phi)$
 Cycloid: trace of point A as ϕ is varied

Constant Controller: [Cycloid.app]
 Options: $\phi = 1.37445$, Step: $\pi/32$

Circle centre $(\phi, 1)$: $x = \phi - \sin\phi$, $y = 1 - \cos\phi$

VOLUME of REVOLUTION

Volume: $33.96\pi \text{ cm}^3$
 Area: 10.90 cm^2

Harry's Goblet: $y = 0.00943x^4 - 0.223x^3 + 1.63x^2 - 3.68x + 2.60$

DIFFERENTIAL EQUATIONS

Equation 1: $y'' + ky = x$
 Equation 2: $y = x - 1$



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USA: Topic Outline for AP CALCULUS AB and BC

with references to Autograph

+ *Topics in ITALICS are in Calculus BC only*



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I. FUNCTIONS, GRAPHS AND LIMITS

Analysis of graphs

- **Emphasis on interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.**



Limits of functions (including one-sided limits)

- **An intuitive understanding of the limiting process**
- Calculating limits using algebra
- **Estimating limits from graphs or tables of data.**



Asymptotic and unbounded behaviour

- **Understanding asymptotes in terms of graphical behaviour**
- **Describing asymptotic behavior in terms of limits involving infinity**
- **Comparing relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth and logarithmic growth).**



Continuity as a property of functions

- An intuitive understanding of continuity
- Understanding continuity in terms of limits
- **Geometric understanding of graphs of continuous functions** (Intermediate Value Theorem and Extreme Value Theorem).



Parametric, polar and vector functions

- + *The analysis of planar curves includes those given in parametric, polar and vector form*



II. DERIVATIVES

Concept of the derivative

- **presented graphically, numerically and analytically**
- **interpreted as an instantaneous rate of change**
- **defined as the limit of the difference quotient**
- **Relationship between differentiability and continuity**



AP Calculus AB and BC

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Derivative at a point

- Slope of curve at a point
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values



Derivative as a function

- Corresponding characteristics of graphs of f and f'
- Link between increasing / decreasing behaviour of f and the sign of f''
- The Mean Value Theorem
- Equations involving derivatives



Second derivatives

- Corresponding characteristics of the graphs of f , f' and f''
- Points of inflection as places where concavity changes



Applications of derivatives

- Analysis of curves including the notions of monotonicity and concavity
- + *Analysis of planar curves, optimization*
- Modeling rates of change
- Implicit differentiation to find the derivative of an inverse function
- Geometric interpretation of differential equations via slope fields
- + *Numerical solution of differential equations using Euler's method, L'Hospital's Rule*



Computation of derivatives

- Including derivatives of power, exponential, logarithmic, trigonometric and inverse trigonometric functions
- Rules for the derivative of sums, products and quotients of functions
- Chain rule and implicit differentiation
- + *Derivatives of parametric, polar and vector functions*



III. INTEGRALS

Interpretations and properties of definite integrals

- Definite integral as a limit of Riemann sums
- Definite integral of the rate of change of a quality over an interval
- Basic properties of definite integrals (additivity and linearity)



Application of integrals

Fundamental Theorem of Calculus

- to evaluate definite integrals
- to represent a particular anti-derivative



AP Calculus AB and BC

Techniques of anti-differentiation

- from derivatives of basic functions
- by substitution, by parts; simple partial fractions
- + Improper integrals (as limits of definite integrals)

Applications of anti-differentiation

- Finding anti-derivatives using initial conditions
- Solving separable differential equations; modelling; exponential growth
- + solving logistic differential equations; use in modelling

Numerical approximations to definite integrals

- Use of Riemann sums and trapezoidal sums to approximate

IV POLYNOMIAL APPROXIMATIONS AND SERIES

Concept of series

- + Convergence, divergence; Series of constants
- + Examples; decimal expansion
- + Geometric Series; Harmonic Series
- + Alternating series; error bound
- + The integral test and its use in testing the convergence of p -series
- + The ratio test for convergence and divergence; comparing series

Taylor series

- + Taylor polynomial approximation; graphical demo of convergence
- + Maclaurin series, formal manipulation of Taylor series centered at $x=a$
- + Maclaurin series for e^x , $\sin x$, $\cos x$, $1/(1-x)$
- + Functions defined by power series
- + Radius and interval of convergence of power series
- + Lagrange error bound for Taylor polynomials

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2D



2D



2D



2D

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