**A MAPPING APPROACH TO PHANTOM GRAPHS.**

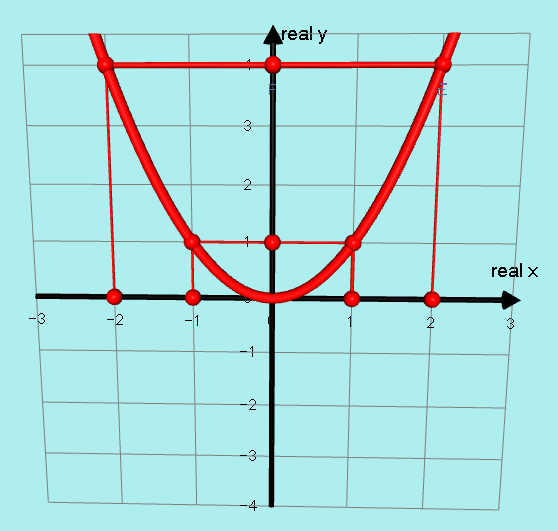
Consider the basic parabola y = x2 drawn on simple x, y axes.

When we plot points we choose x values and calculate corresponding y values.

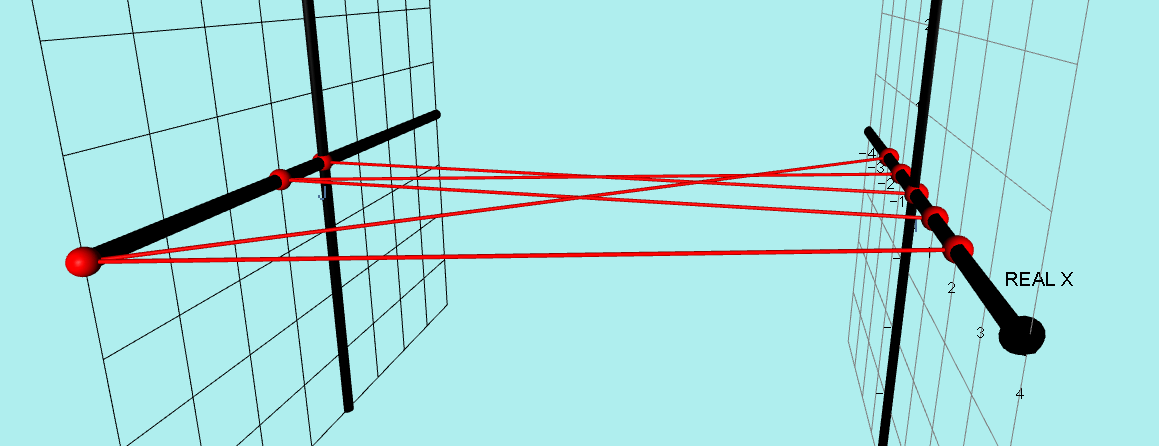
x = 1 so y = 12 = 1 then plot (1, 1)

x = 2 so y = 22 = 4 then plot (2, 4)

as shown below.



I want to think of this as mapping x values from an x axis onto y values on a parallel y axis as below:



1

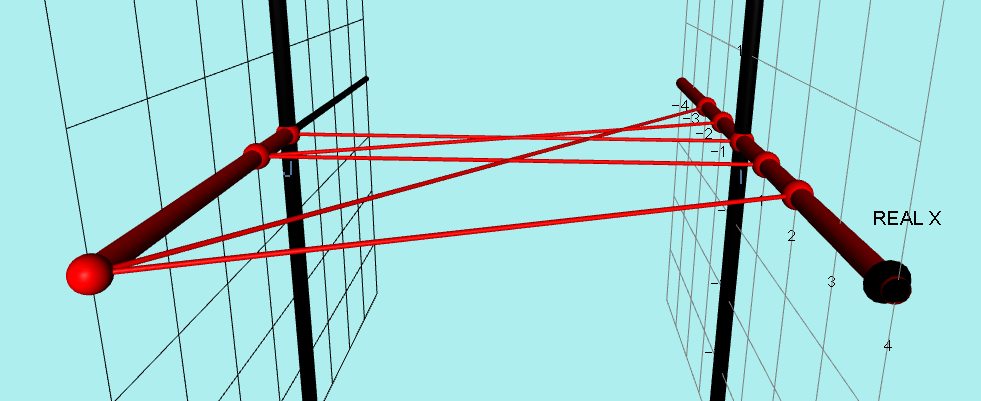
2

3

4

REAL Y

Obviously, positive and negative x values map onto only positive y values.

I will show this by colouring the **whole x axis** **RED** but only the **positive y axis** 

1

2

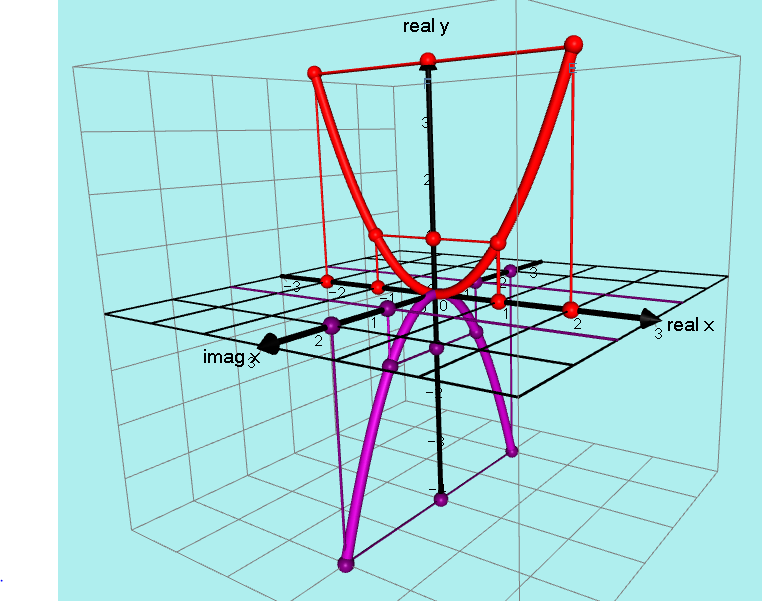
3

4

REAL Y

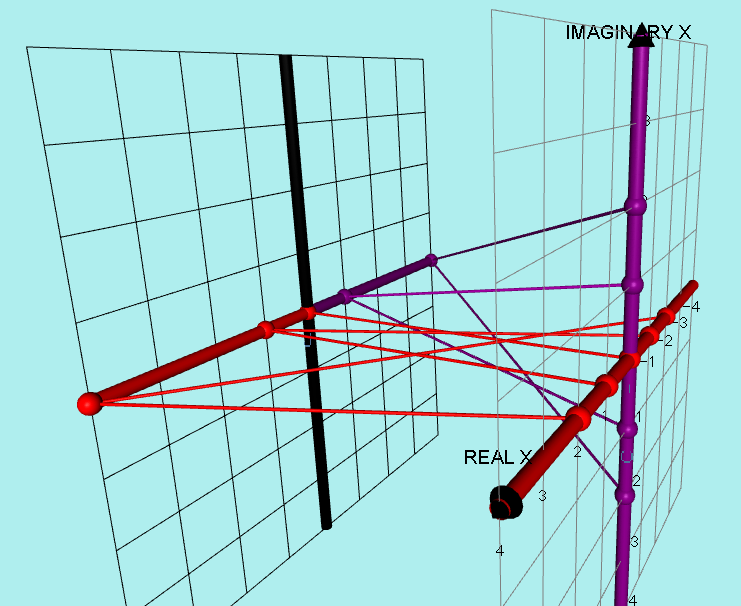
Of course the basis of phantom graphs is that some complex x values do produce real y values when substituted in the graph’s equation.

In this case, if ***x = i*** then ***y = i2 = – 1*** and if ***x = 2i*** then ***y = i2 = – 4 as below:***



The REAL points are **RED** and the IMAGINARY points are **PURPLE.**

To demonstrate this as a mapping process, we need an **x plane** mapping onto just a **y axis**.



**REAL Y**

**IMAGINARY Y**

-4

-3

-2

-1

1

2

3

4

Notice that the whole **REAL x axis** maps onto only the **positive y values** and

the whole **IMAGINARY x axis** maps onto only the **negative y values.**

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**Now we come to a whole new stage where we consider what happens to the other complex x values when we substitute them into the equation y = x2**

Just as both **x = 2** and **x = – 2** map onto **y = +4** and **x = 2*i*** and

**x = – 2*i*** both map onto **y = – 4, the same thing happens with pairs of complex x values. They occur in pairs.**

**x = 1 + i maps onto y = (1 + i)2 = 2i**

**x = –1 – i maps onto y = (–1 – i)2 = 2i**

**x = 2 + i maps onto y = (2 + i)2 = 3 + 4i**

**x = –2 – i maps onto y = (–2 – i)2 = 3 + 4i**

**x = 1 + 2i maps onto y = (1 + 2i)2 = –3 + 4i**

**x = –1 – 2i maps onto y = (–1 – 2i)2 = –3 + 4i**

**x = 1 – i maps onto y = (1 – i)2 = –2i**

**x = –1 + i maps onto y = (–1 + i)2 = –2i**

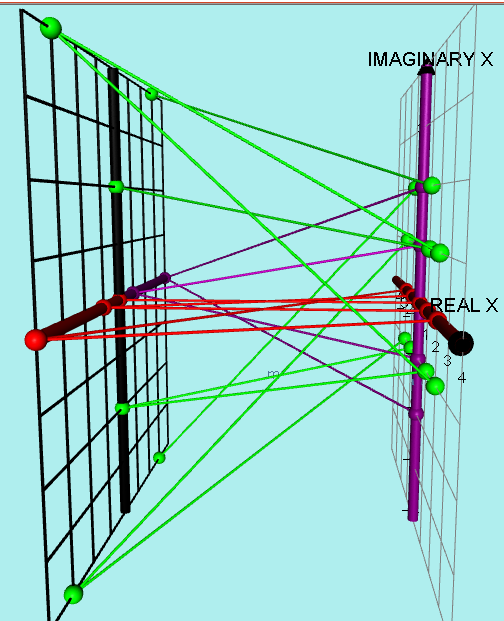
**x = 1 – 2i maps onto y = (1 – 2i)2 = –3 – 4i**

**x = –1 + 2i maps onto y = (–1 + 2i)2 = –3 – 4i**

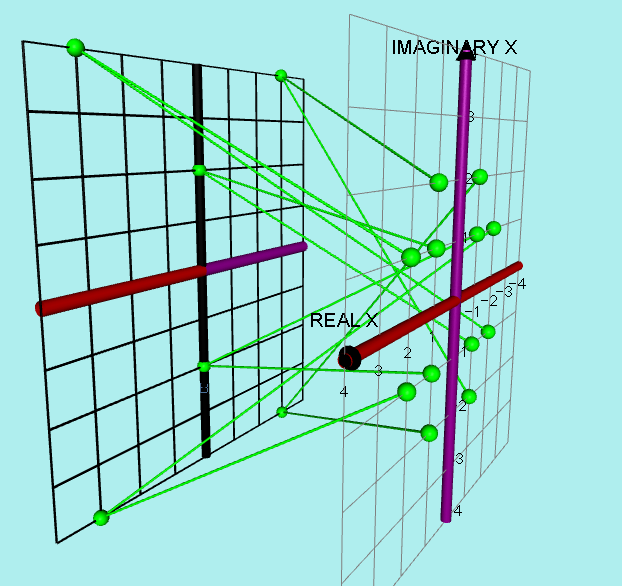
**x = 2 – i maps onto y = (2 – i)2 = 3 – 4i**

**x = –2 + i maps onto y = (–2 + i)2 = 3 – 4i**

**I have put these points on as GREEN DOTS AND LINES.**



**To make this a little bit clearer I will take off the red and purple lines and dots.**



**(–3, + 4i)**

**(3, + 4i)**

**(3, – 4i)**

**REAL Y**

**IMAGINARY Y**

Notice that the **12 points** chosen

on the **complex x plane** only

mapped onto **6 points** on the

**complex y plane**.

At this stage there does not seem to be a clear pattern forming from these complex y points.

I decided to add just a few more:

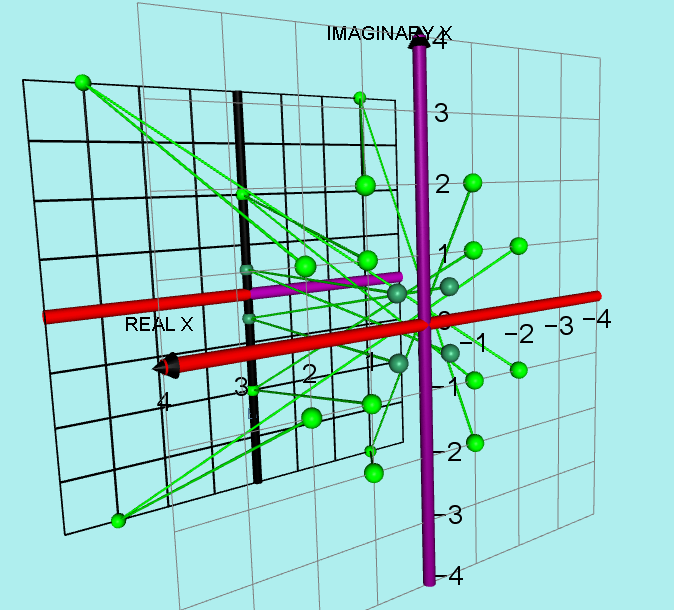
**x = ½ + ½ *i* maps onto y = (½ + ½ *i*)2 = 0 + *i***

**x = –½ – ½ *i* maps onto y = (–½ – ½ *i*)2 = 0 + *i***

**x = –½ + ½ *i* maps onto y = (–½ + ½ *i*)2 = 0 – *i***

**x = +½ – ½ *i* maps onto y = (+½ – ½ *i*)2 = 0 – *i***

The above 4 point are coloured a **darker green.**



**IMAGINARY Y**

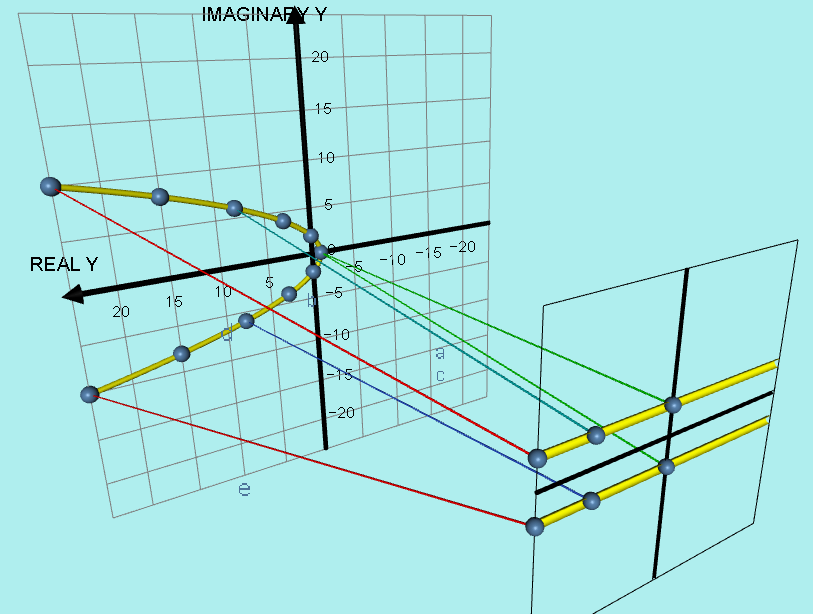
**REAL Y**

It should be noted that, for example, if **x = 2 + *i*** then **y = 3 + 4*i***

so we would need two dimensions to plot **x = 2 + *i*** as **(2, 1)** and two further dimensions to plot **y = 3 + 4*i*** as **(3, 4)** which of course implies FOUR DIMENSIONAL SPACE so the above mapping cannot be represented as an actual 4D “curve” or “surface” since we can only function in 3D space.

However, I did find some **surprising patterns** when mapping sets of linear points on the complex x plane onto the complex y plane!

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***a ± 1i*** MAP ONTO THE COMPLEX Y PLANE AFTER y = x2



***1i***

***–1i***

***y = 24 +10i***

***y = 24 – 10i***

***x = 5 – i x = 3 – i x = 0 - i***

IMAG x

Real x

I used complex ***x*** values in the form of ***± a ± 1i***  as shown by the yellow lines above on the complex x plane and transformed them using the equation ***y = x2***

onto a complex ***y*** plane.

As you can see the result was complex **y** values forming the parabola, given by the equation **y = t2 – 1** using **t** as the variable for the imaginary ***y*** coordinates.

**4**

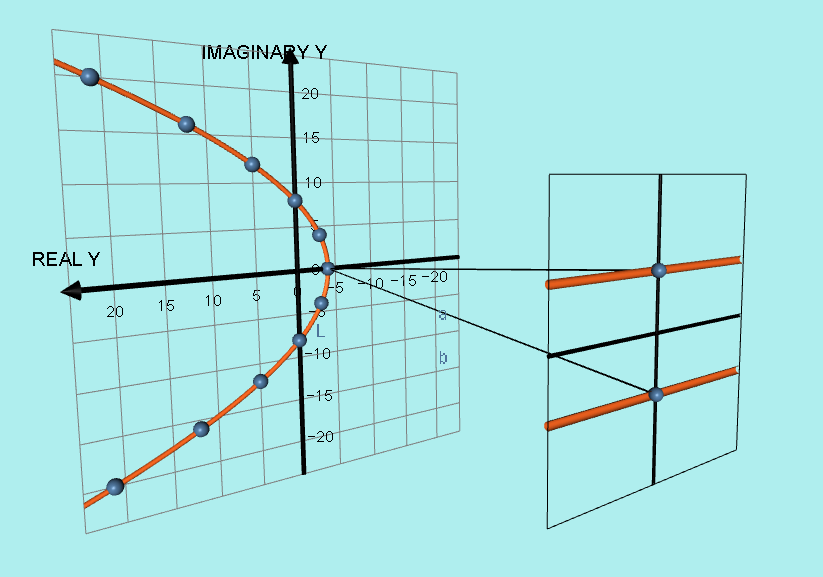
**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**I decided to repeat the investigation for complex x values in the forms**

**x = *± a ± 2i*** and x = ***± a ± 3i***

In each case, keeping the imaginary part constant and varying the real part.

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***a ± 2i*** MAP ONTO THE COMPLEX Y PLANE AFTER ***y = x2***



**2i**

**– 2i**

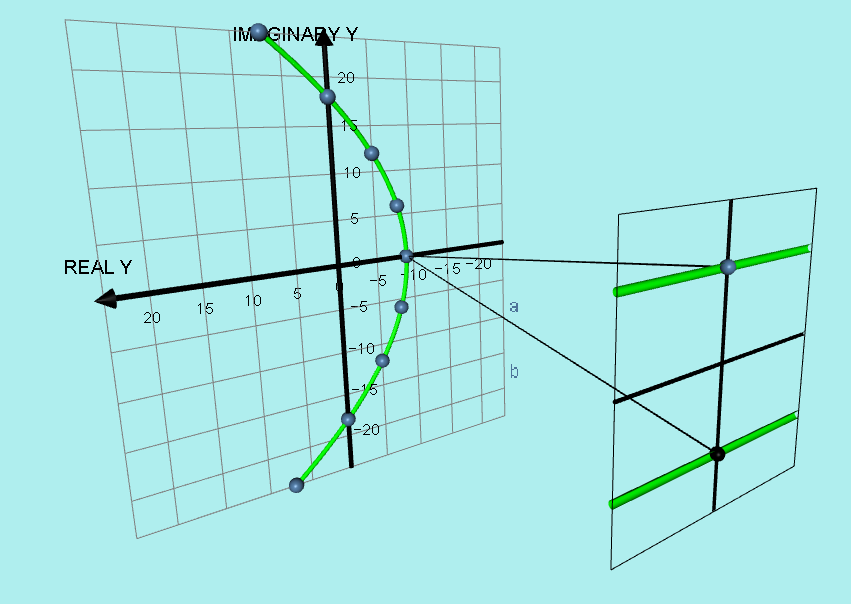
Imag x

Real x

**This time the equation of the parabola formed was y = t2 – 4**

**16**

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***a ± 3i*** MAP ONTO THE COMPLEX Y PLANE AFTER ***y = x2***



**3i**

**– 3i**

Real x

Imag x

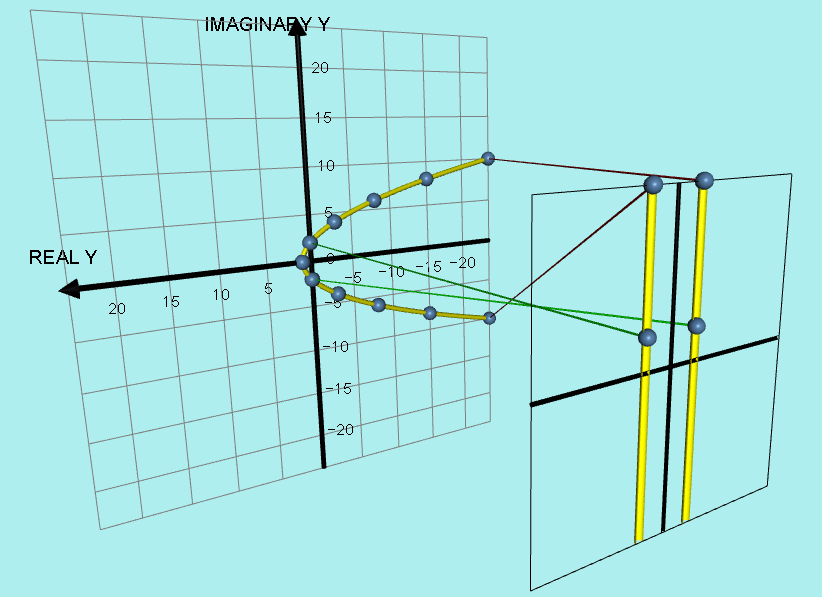
**This time the equation of the parabola formed was y = t2 – 9**

**36**

**Next I decided to transform numbers of the form *x = (1 + bi)***

**Keeping the real part constant and varying the imaginary part.**

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***1 ± 1i*** MAP ONTO THE COMPLEX Y PLANE AFTER ***y = x2***



–1

1

Imag x

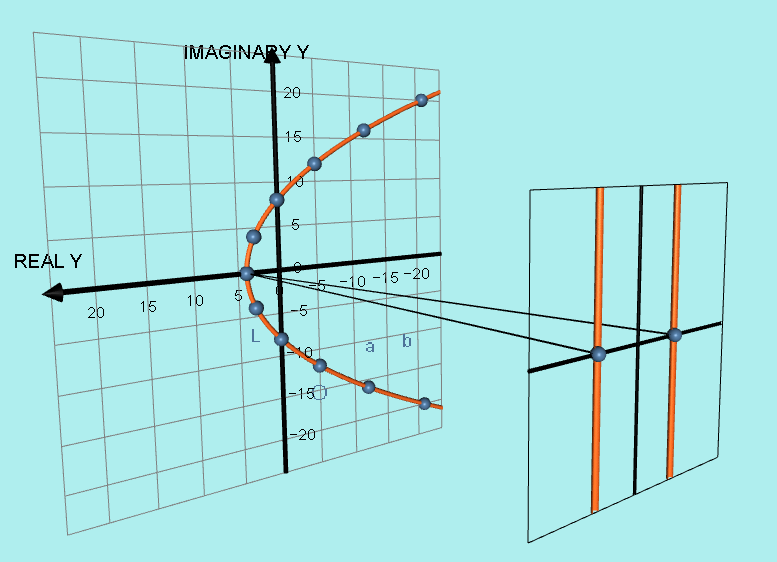
Real x

**This is just a reflection of the transformation of x values of the form *a + 1i***

The parabola of complex y values has the equation **y = – t2 + 1**

**4**

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***1 ± 2i*** MAP ONTO THE COMPLEX Y PLANE AFTER ***y = x2***



–2

2

Imag x

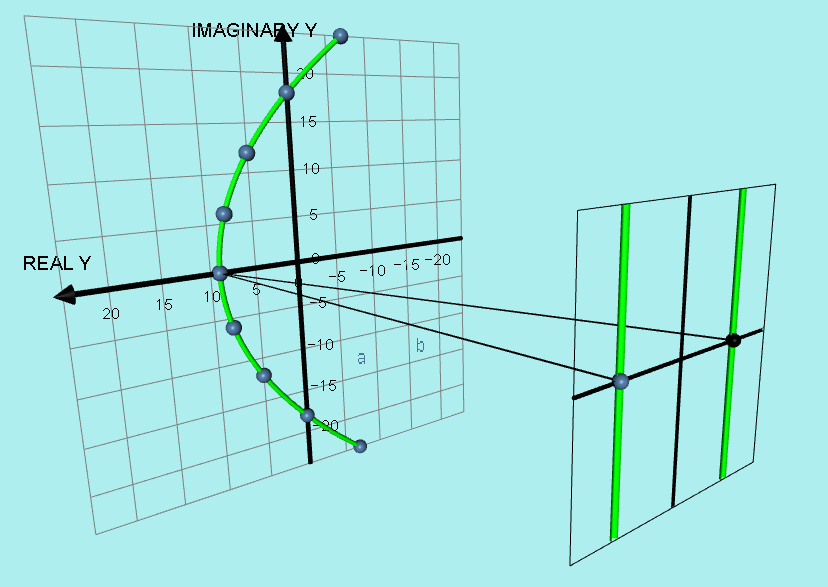
Real x

**This is just a reflection of the transformation of x values of the form *a + 2i***

The parabola of complex y values has the equation **y = – t2 + 4**

**16**

MAPPING SHOWING HOW POINTS IN THE COMPLEX X PLANE OF THE FORM ***1 ± 3i*** MAP ONTO THE COMPLEX Y PLANE AFTER ***y = x2***



3

–3

Imag x

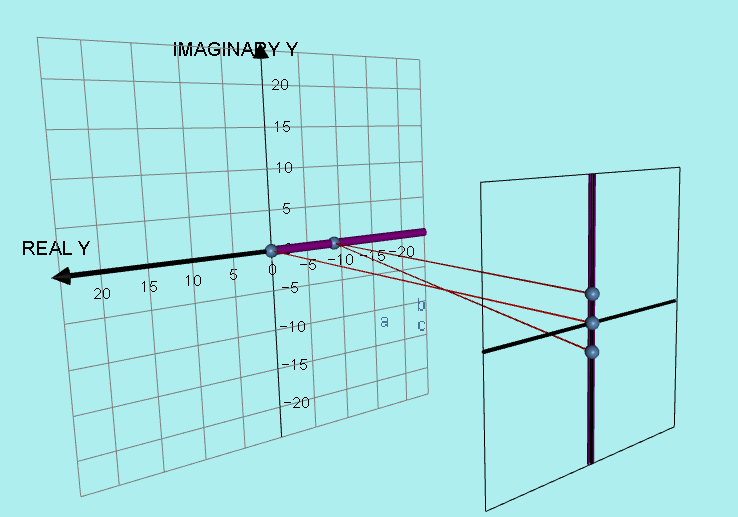
Real x

**This is just a reflection of the transformation of x values of the form *a + 3i***

The parabola of complex y values has the equation **y = – t2 + 9**

**36**

One final interesting point is that if I transform x = 0 + bi all the points become totally REAL on the complex y plane which is the very basic form of *phantom* graphs!



3

–3

Imag x

Real x