ADVANCED AUTOGRAPH

ADVANCED AUTOGRAPH WEBINAR: four 1½-hour sessions October/November 2020 by Douglas Butler (iCT Training Centre, Oundle) and Rob Smith (La Salle Education)

SESSION 9: Using the Attribute Tools
Wednesday 7 October 2020, 7:30-9pm

1. Heron's Formula (Area of a triangle)
2. The Incircle of a 3-4-5 Triangle
3. Mean Value of a function/Mean Value Theorem
4. Exploring inverse functions, and inverse trig
5. Steering Wheel simulation

SESSION 10: Coordinate Geometry Tools
Wednesday 14 October 2020, 7:30-9pm

1. The many representations of the circle
2. Conic sections (3D) and Star-Trek
3. Matrix transformations (2D and 3D)
4. Parametric plotting: projectiles
5. The Valentine heart!

SESSION 11: Calculus and 3D Tools
Wednesday 28 October 2020, 7:30-9pm

1. Introducing 'e' and why is $\int (1/x)dx = \ln x + c$
2. 1^{st} Order Differential Eqns: terminal velocity
3. Dot product (2D and 3D)
4. 3D vectors, lines and planes
5. Volume of revolution

SESSION 12: Data and Probability Tools
Wednesday 4 November 2020, 7:30-9pm

1. Fitting models to data
2. Spearman' Rank/Correlation Coefficient
3. x = g(x) iteration
4. Binomial and Poisson probabilities
5. Normal distribution: hypothesis testing

EXTENSION TOPICS

1. Complex Numbers and the Argand Diagram
2. Polar plotting
3. 2^{nd} order Differential Equations/SHM
4. Arc length and surface of revolution
5. Construction of a Cycloid

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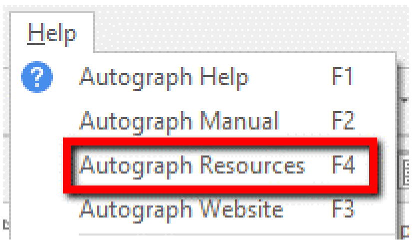
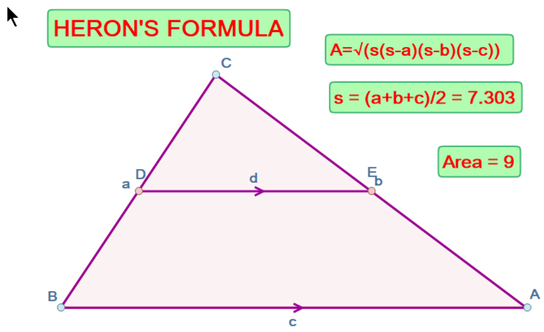


Autograph Resources: Press F4 ->
www.tsm-resources.com
Complete Mathematics Webinar Documentation and .agg files:
https://courses.completemaths.com/autograph-advanced-lesson-ideas

SESSION 9: Using the Attribute Tools

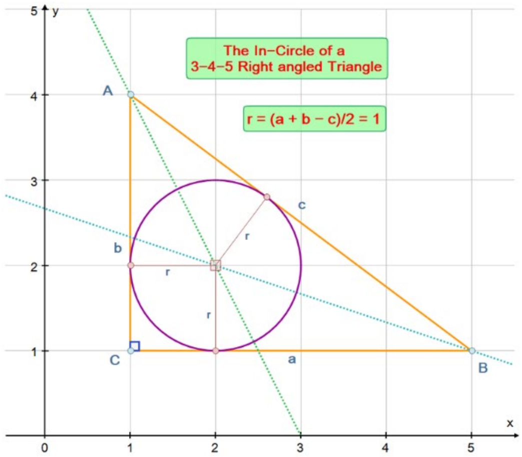
1. HERON'S FORMULA (Area of a triangle)



Clear axes. Label the triangle A-B-D with sides a-b-c
Manage the constants 'a', 'b' and 'c' accordingly
Use the calculator to evaluate 's' = (a+b+c)/2
Manage the constant 's' to be the semi-perimeter.
Use the calculator to evaluate √(s(s-a)(s-b)(s-c))

Autograph file: 1. Heron.agg

2. THE INCIRCLE OF A 3-4-5 TRIANGLE



Axes menu: set x- and y-snaps to 1 and 1
Create a triangle with sides 3-4-5

Select 3 points ->
a. Create Shaded Area
(also offers Perimeter and Area as attributes)
b. Then angle bisector: again with the next three.

Select the two bisectors
Edit Draw options -> dashed
Intersection mode: mark the intersection
This is the centre of the in-circle, radius 1

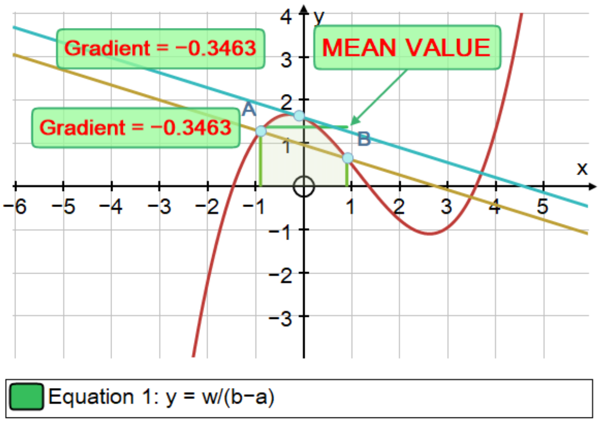
Does r = (a+b-c)/2?
What about a 5-12-13 triangle?
Can you prove this formula?

Autograph file: 2. 3-4-5 Triangle.agg

Excel file: PythagTriples.xls

3. MEAN VALUE OF A FUNCTION/MEAN VALUE THEOREM

Create a cubic from four points
Place two points on curve 'A' and 'B'
Use "Manage Constants" to associate 'a' and 'b' with the x-coordinates of these two points.
Select the two points A and B: straight line
Then Create -> Area.
Use "Manage Constants" to set w = this area



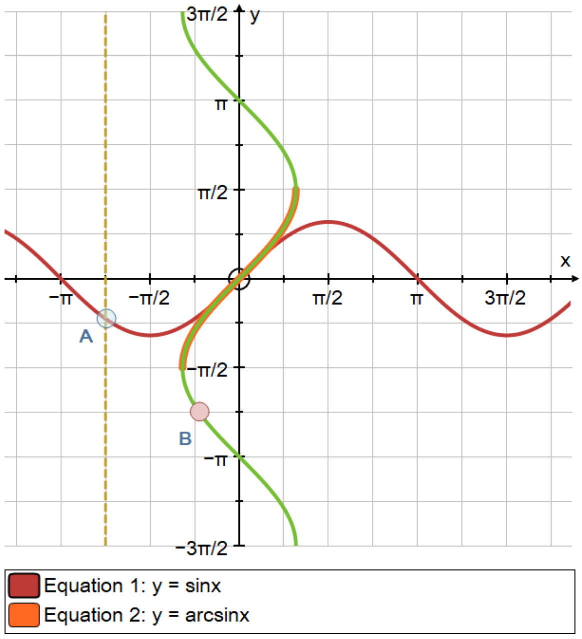
a. The Mean Value of the function is y = w/(b-a): use start-up options to set the domain from 'a' to 'b'

b. The Mean Value Theorem states that there is a tangent parallel to a chord AB within a < x < b. Place a point on the curve - draw a tangent. Use the Calculator to display the gradient of both lines. Use arrow keys and CTRL and SHIFT to equalise the gradients (confirmed using the Parallel test)

Autograph file: 3. Mean Value.agg

4. EXPLORING THE INVERSE TRIG FUNCTION

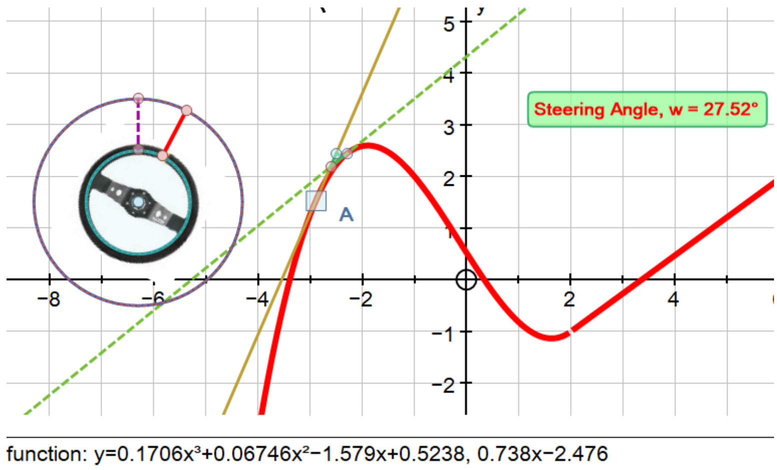
Use 'Edit Axes' to set up π-scales on both axes, with 'Equal Aspect' on. Draw y = sinx
Place a point 'A' on this curve
Use "XY-Attribute Point" to create a second point 'B' which takes its 'x' coordinate from the 'y' of 'A' and its 'y' coordinate from the 'x' of 'A'.



Move 'A' around on y = sinx and observe 'B'
Plot x = siny. Draw a vertical line through 'A' and discuss the need for a one-to-one relationship with an inverse function.
Plot y = arcsinx and discuss its domain and range.

Autograph file: 4. Inverse Trig.agg

5. A STEERING WHEEL SIMULATION!

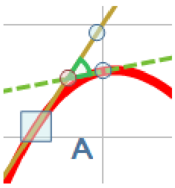


To create the roadway:
Select 4 points to create a cubic graph.
Put a point at x = 2 and draw tangent

Select one of the points defining the roadway, and the tangent. The equations of both will appear in the STATUS BOX at the bottom.
Use Page => Copy Status Box, then Enter a new Equation by pasting and trimming to the form of a PIECEWISE equation: y = f(x), g(x), then visit 'Startup Options' to set the x-dividers: −4, 2, 10
Set the Draw options for this graph to thick, 4½pt

Put a point 'A' on this graph, and drag it around to make sure is goes all the way along.

Draw a tangent at 'A': Use Manage Constants to assign the letter 'a' to the x-coordinate of 'A'
Now to create the second tangent which will predict the wheel turning.
Enter a new point on the curve at x = a+k, where we can alter 'k' as required.
Draw a second tangent, and mark the intersection of the two tangents



Use Edit Draw Options to make the second tangent dashed,
Place a point on the first tangent and select three to find the angle between the two tangents – this is the angle to be used by the steering wheel.
Important, tick the options "allow Reflex Angle" and "Display angle to 1 d.p.
Use Manage Constants to assign the letter 'w' to this angle.

Use the Constant Controller to set 'k' to around 0.4

Drag the Steering when image on. Place a point somewhere near it, select the point and the wheel to create a rotation through angle 'w'. Hide the original wheel, and move the point to where the centre of the wheel was. Move the point 'A' – enjoy!!

Optional extras: create two circles and control line segments. This will involve a vertical line through the when centre, and two circles, and two intersection points, hence the vertical line segment. Hide the original vertical line, and make the line segment dashed using Edit Draw Options. Select the higher point and the centre and rotate the higher point through w°. Join to the centre, find the intersection with the inner circle, hence the final line segment (delete the line).

Autograph file: 5. steering.agg
Image files: 5. steering.jpg and 5. car.jpg

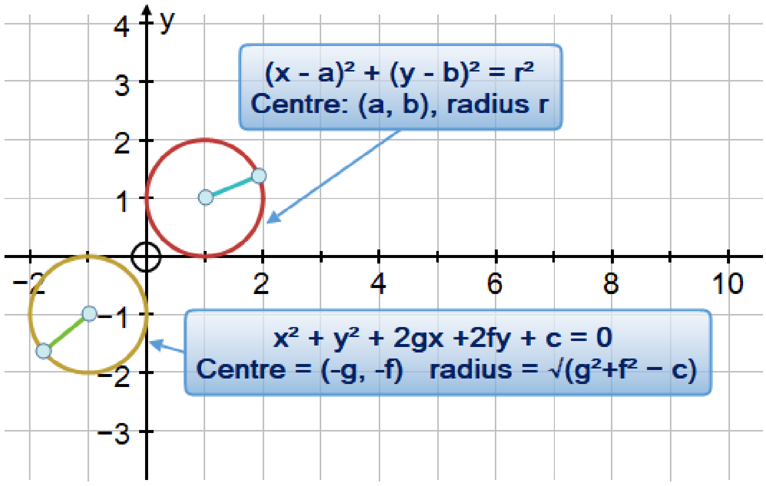
SESSION 10: Coordinate Geometry Tools

1. THE MANY REPRESENTATIONS OF THE CIRCLE

1a. Equation of a circle
$(x - a)^2 + (y - b)^2 = r^2$
Centre: (a, b), radius r
Use the constant controller

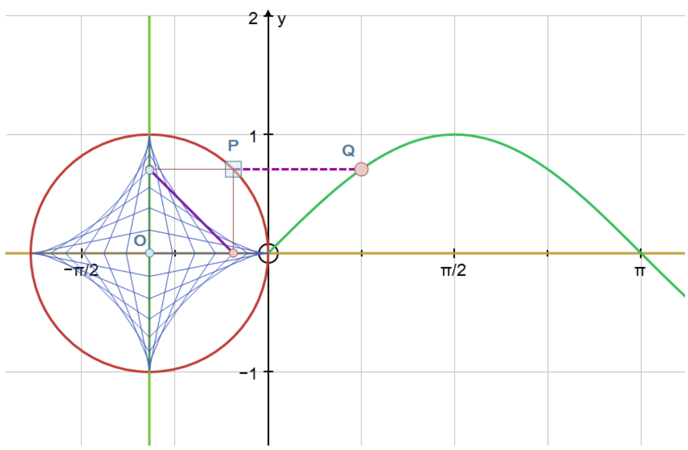
1b. General form for a circle
$x^2 + y^2 + 2gx + 2fy + c = 0$
Centre = (-g, -f)   radius = $\sqrt{(g^2+f^2 - c)}$
Use the constant controller

Autograph file: 1. circles1.agg



1c. Unit circle and sinx, using Locus
Draw a circle with centre (-1,0) radius 1.
Place and select a point 'P' on the circle.
This will have three components, x, y,t, where 't' is the angle ante-clockwise from the origin.

Create a new XY Attribute Point 'Q' using 't' as the new 'x', and 'y' as the new 'y'.
Move 'P' about.
Select 'P' and 'Q' and Create -> Locus
Rescale the 'x' axis with π scales.
Check that Equal Aspect is on.



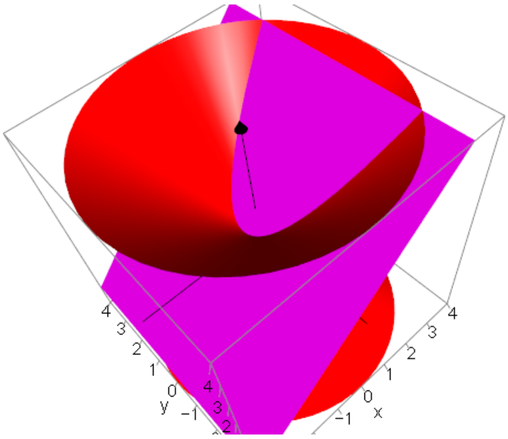
Draw a vertical line through O, and enter y=0 to draw the x-axis
Select 'P' and each of these toe lines to draw the two closest points.
Select them and draw the line segment connecting them.
Watch what happens to the line segment as you pull 'P' round the circle.
Finally select 'P' and the line segment and Create -> Locus
0 to 2π in steps of π/16

Autograph file: 1. circles2.agg

You can also create circles:	1st order Differential equation:	$y' = -y/x$	1.circles3.agg
	Parametric equation:	$x = \sin t$, $y = \cos t$	1.circles4.agg

2. COORDINATE GEOMETRY IN 3D

2a. Conic Sections



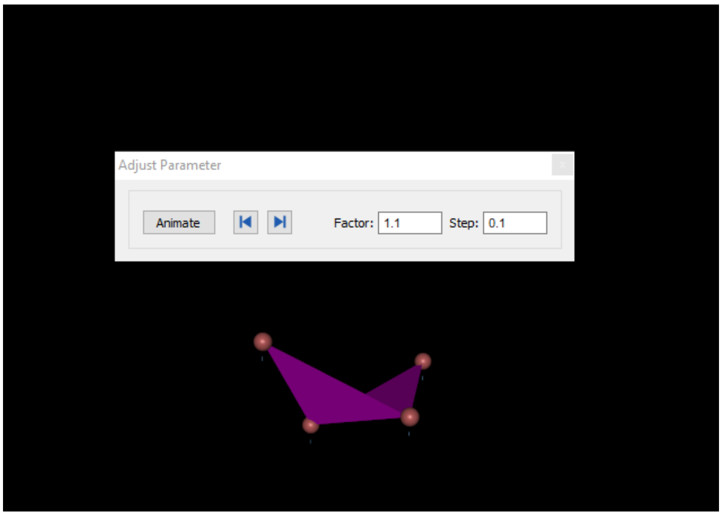
Enter a cone: $x^2 + y^2 = z^2$, or $r = z$
Enter a plane: $z = ax + b$

Drag rotate the scene.
CTRL-up and down to zoom in
Use the Constant controller to vary 'a' to see:
Hyperbola, Parabola, Ellipse, Circle

Autograph file: 2. conics.agg

CONIC EQUATIONS Eccentricity, ε:		Cartesian	Parametric	Polar (focus at O, LR at ℓ)
$\varepsilon = 0$	**Circle**	$x^2 + y^2 = r^2$	$x = \cos\theta,\ y = \sin\theta$	$r = a$
$\varepsilon < 1$	**Ellipse**	$x^2/a^2 + y^2/b^2 = 1$	$x = a\cos\theta,\ y = b\sin\theta$	$r = \varepsilon\ell/(1 - \varepsilon\cos\theta)$
$\varepsilon = 1$	**Parabola**	$y^2 = 4ax$	$x = at^2,\ y = 2at$	$r = 1/(1 - \cos\theta)$
$\varepsilon > 1$	**Hyperbola**	$x^2/a^2 - y^2/b^2 = 1$	$x = a\sec\theta,\ y = b\tan\theta$	$r = \varepsilon\ell/(1 - \varepsilon\cos\theta)$
$\varepsilon = \sqrt{2}$	**Rect Hyp**	$xy = c^2$	$x = ct,\ y = c/t$	$r = \sqrt{2}\ell/(1 - \sqrt{2}\cos\theta)$

2b. Star-Trek Video



Play the video **2. startrek.mp4**
Load the file **2. startrek1.agg**

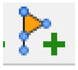
With 5 points to make the spacecraft.
Select 3 for the right wing "Group to shape"
Select the other 3 "Goup to Shape"
Select the back point and the shape:
Transformation -> Enlargement, factor 2

Hide the original shape and the back point.
Select the transformed shape, open the
Animation Controller, wind it back to 1

Open Edit Axes – Options -> Axes
Choose "None" and untick "Show Bounding Box"
"Lights" (switch to BLACK background) – "Camera" – "Action"
Play the music **2. startrek.mp3**, and animate the spacecraft.

Autograph file: 2. startrek2.agg

3. MATRIX TRANSFORMATIONS (2D)

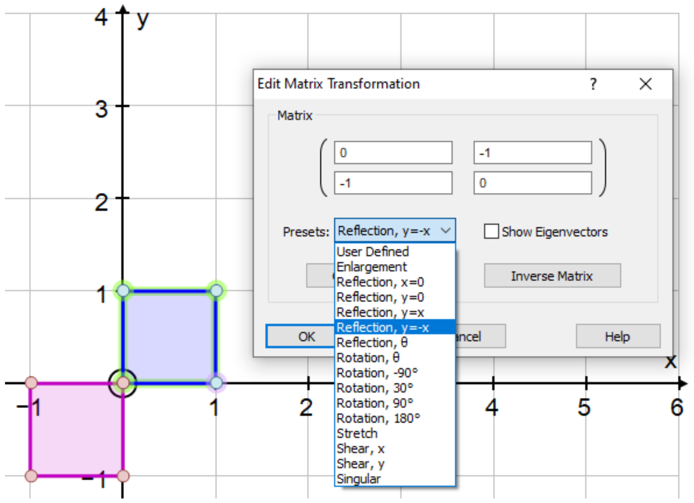

Add shape and choose Unit Square
Select the square and Transform
-> Matrix Transformation

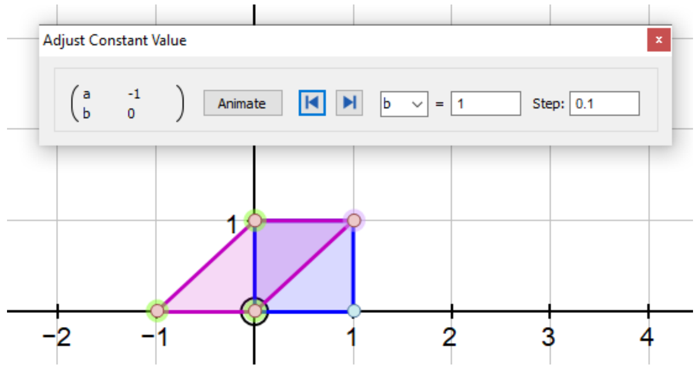
You can enter your own numbers or Constants into the matrix, remembering The left column is the transformation of (1, 0) and the right transforms (0, 1).

There is a long list of user-defined pre-sets. Press the Inverse Button to show the inverse.

If the matrix includes constants, use the Animation Controller to vary the values.

If the matrix does not include constants, the Animation Controller applies the matrix over and over, or its inverse.



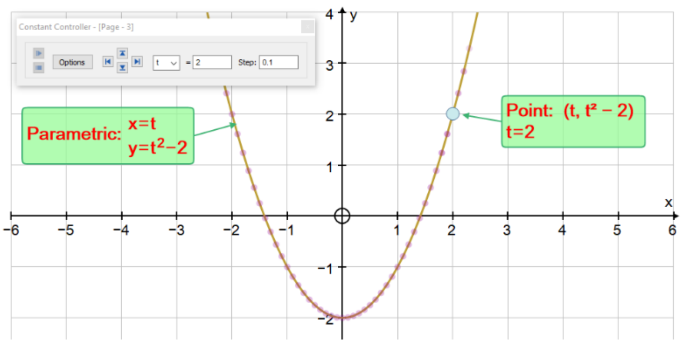


4. PARAMETRIC PLOTTING: PROJECTILES

4a. Introducing parametric plotting

Enter a point (t, $t^2 - 2$). Use constant controller to more the point around, and turn on Trace.
Enter the parametric equation x = t, y = $t^2 - 2$

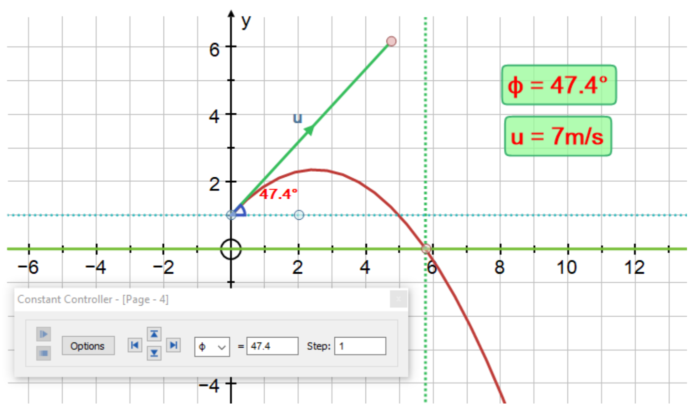
Use the Text Box to display information.
Autograph file: 4. parametric1.agg



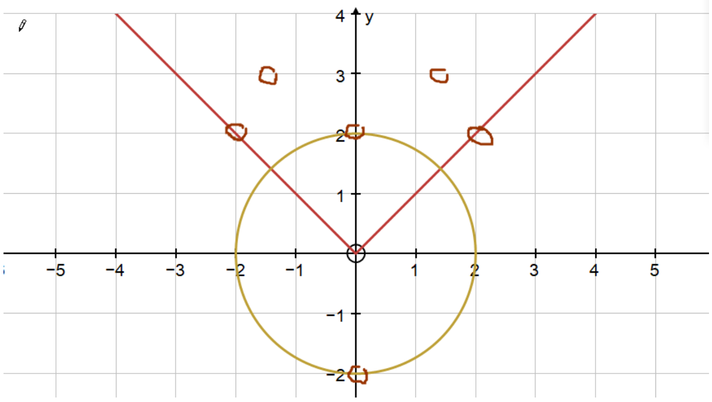
4b. Projectile Motion

In degrees mode
Enter x = (ucosϕ)t, y = h + (usinϕ)t – ½gt^2
Start-up options -> manual 0, 10, 0.1
Constants: g=9.81, h = 1, u = 7, ϕ = 45
Enter y = 0; mark the intersection.
Use constant controller to vary u, h and ϕ

Autograph file: 4. parametric2.agg

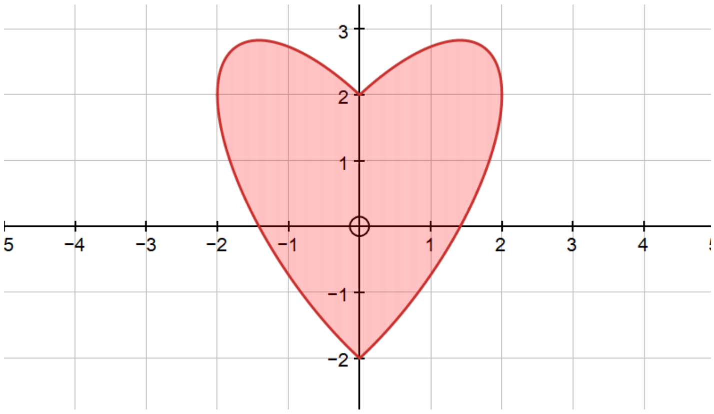


5. THE VALENTINE HEART!



Slow plot on. Equal aspect on.
Enter $y = |x|$
Enter $y = y = \pm\sqrt{(r^2 - x^2)}$ with $r = 2$

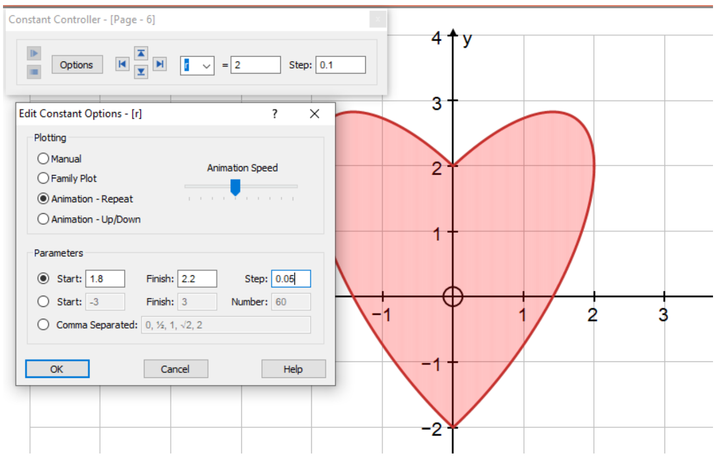
Discuss adding these two functions,
Using the scribble tool.



New 2D page, enter $y = |x| \pm \sqrt{(r^2 - x^2)}$
Again with $r = 2$.
This '±' plot is in two halves.
To fill the 'heart' area:

Select the top half then the bottom half.
Create -> area
- General
- with Start Point –r, End Point r

Change the colour using the colour picker



To animate the heart
Constant controller -> Options

Animation – repeat
1.8 – 2.2 – 0.05

Click on "No axes"
Enjoy!

Autograph file: 5. Valentine.agg

Complete Mathematics

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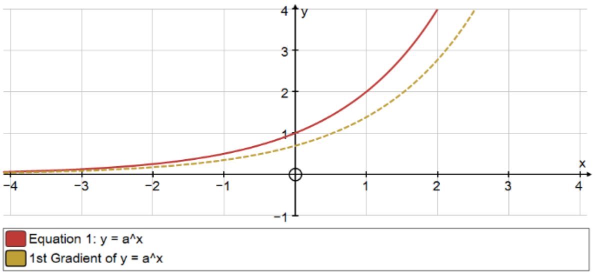
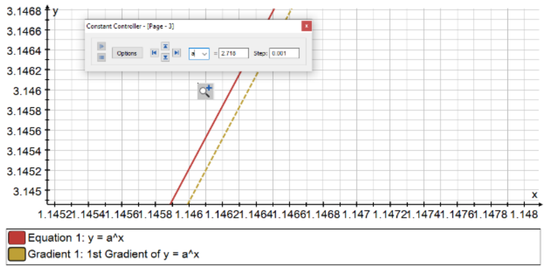
SESSION 11: Calculus and 3D Tools

1. CALCULUS TOOLS

1a. Introducing 'e'



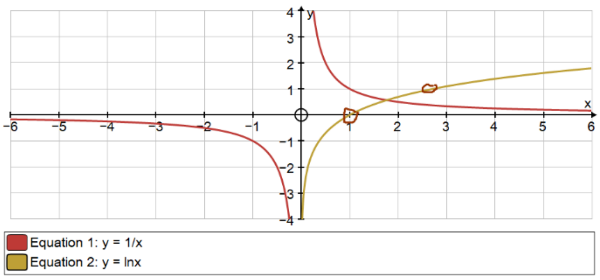
Refer to TSM Lists: [Powers of 2]
Plot **y = 2^x** with slow plot on, and scribble prediction
Edit this equation to **y = a^x** with a = 2
Create the Gradient Function
Use Constant Controller and Zoom to establish a value for e, the function which is equal to its own derivative. Watch out for local straightness.



You can confirm this by entering the differential equation: **y' = y**

Autograph file: 1a. exp.agg

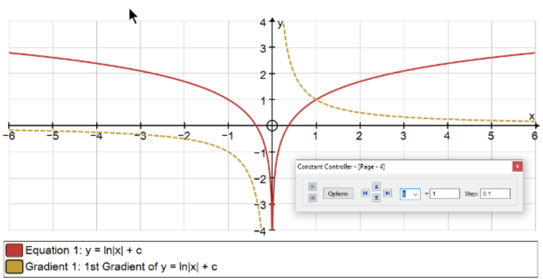
1b. Why is ∫ (1/x)dx = lnx + c ?



First, the theory:

y = lnx => **x = e^y**
=> **dx/dy = e^y = x**
=> **dy/dx = 1/x**
=> **∫(1/x)dx = y = lnx + c**

With slow plot on: plot **y = 1/x** then
y = ln(x) then gradient function
y = ln(-x) then gradient function
y = ln|x| then gradient function
y = ln|x| + c then gradient function



Plot **y = ln|x| + c** and its gradient function
Vary c

Autograph file: 1b. lnx.agg

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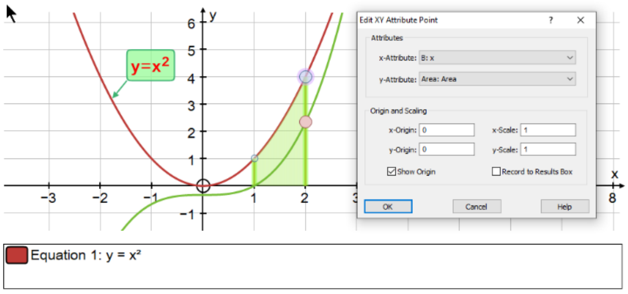
1c. The integral function

Plot **$y = x^2$**
Add and select points on this graph
at x = 1 and x = 2, create -> Area
Select the point at x=2 and the area
XY attribute point, then locus
Try moving the first point away from x = 1



Autograph file: 1c. int-x.agg

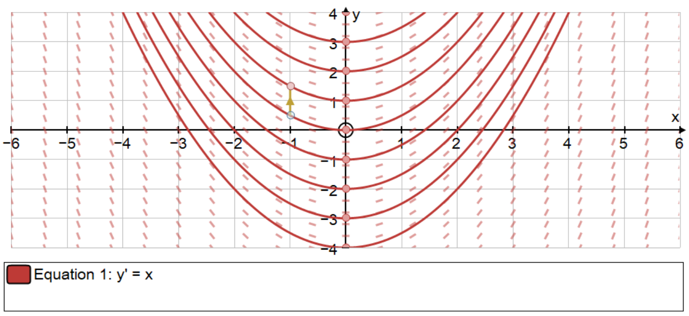
2. 1st ORDER DIFFERENTIAL EQUATIONS:

2a. A 'bottom -up' approach

First, Slow plot is useful, and to set up the Start-up Options:
- Manual – solutions start form where you click. Forwards then back
- Point – solutions start from a selected point, which can be moved around
- Point Set: can create a set of start point on a line, eg the y-axis

A 'bottom up' approach can be useful. Enter:

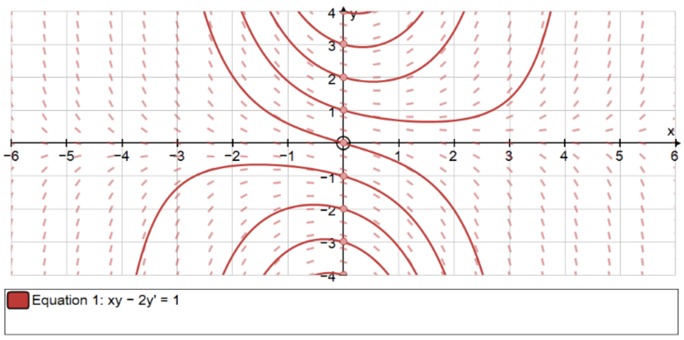
$y' = 0$
$y' = 1$
$y' = x$
$y' = y$
$y' = -y$
$y' + y = 0$
$y' + y = 1$
$y' + y = x$
$y' + y = \sin x$



Autograph file: 2a. de.agg

Explore other possibilities:
$y' = y/x$ [straight lines]
$y' = -x/y$ [circles]

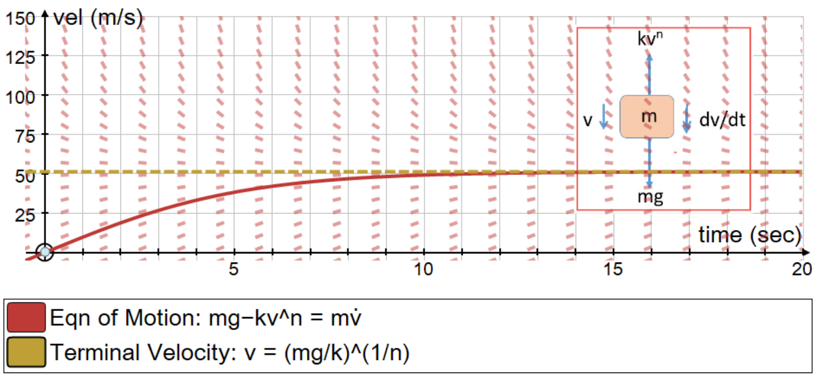
Look around text books for interesting DEs:
Eg **$xy - 2y' = 1$**



2b. Terminal Velocity

In edit Axes, set up variables v, t and axes labels.
Place and select a point at (0,0)
Slow plot on.

By Newton's 2nd law, enter
mg−kv^n = mv'
Edit constants:
g = 9.81, k = 0.3, m = 80, n = 2



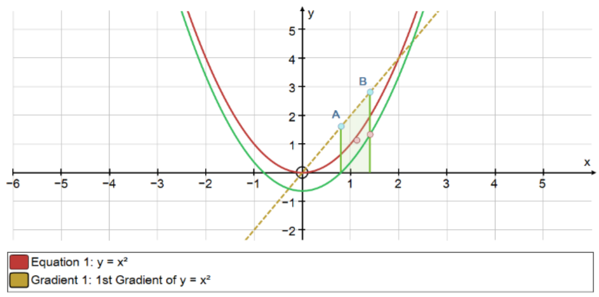
Observe the solution levelling off.
At terminal velocity, the acceleration is zero, so mg = kv^n
Now enter: v = (mg/k)^(1/n)

Autograph file: 2b. Terminal.agg Vary m?

3. THE FUNDAMENTAL THEOREM OF CALCULUS

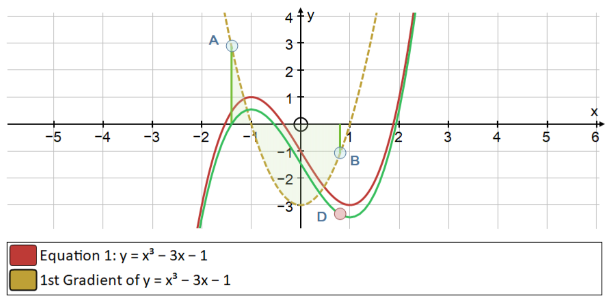
Draw $y = x^2$ and it gradient function

Place and select two points 'A' and 'B' on the Gradient Function
Create -> Area (choose 'General')
Select 'B' and the Area and "XY Attribute Point"
This is 'D'.
Move 'B' along the curve.



Select 'B' and 'D' -> Create -> Locus
Manipulate 'A' until the curves match:
then it is clear that the integral of the differential is back where you started!

Repeat with my favourite cubic: $y = x^3 - 3x - 1$

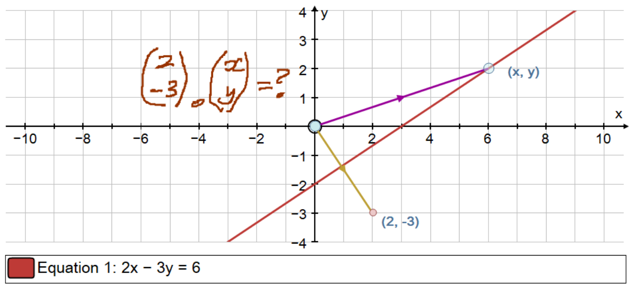


Autograph file: 3a. fundamental.agg
3b. fundamental.agg

4. LINES AND PLANES

4a. Dot Product (2D and 3D)

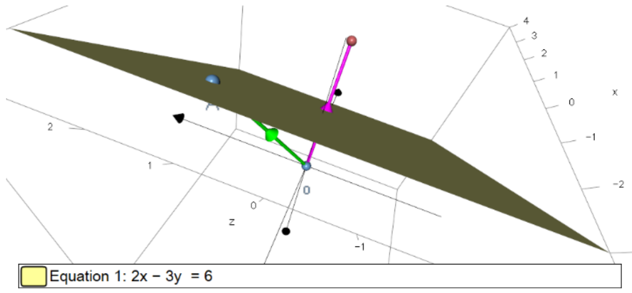
Set slow plot and equal aspect
Enter 2x – 3y = 6 and a point at (0, 0) and a point on the line, 'A'. Show that the vector [2, –3] is perpendicular to the line.



Draw vector [x, y] from O to A. The dot product: [2, –3] **.** [x, y]: consider the algebraic and the geometric definition. Note that the value of the dot product is constant, regardless of the position of 'A'.

So the equation of the line ax + by = c is given by [a,b] **.** [x,y] = constant.

Autograph file: 4a. dot 2D.agg

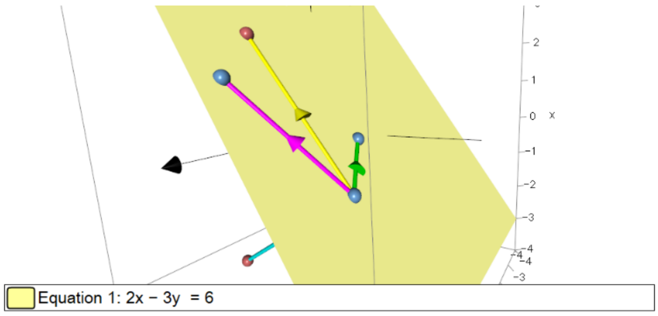


Likewise, in 3D, the equation of a plane ax + by + cz = d, given by [a,b,c] **.** [x,y,z] = constant, the vector [a, b, c] is perpendicular to the plane.

Autograph file: 4a. dot 3D.agg

4b. Cross Product (3D)

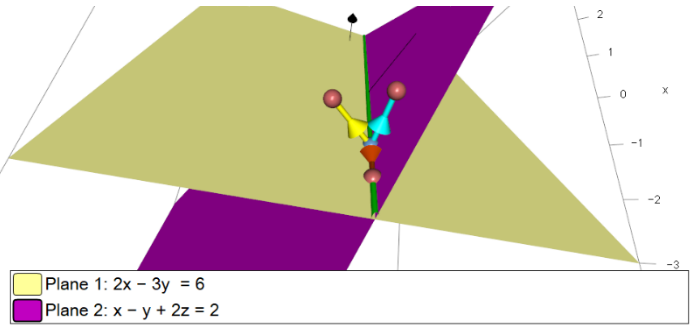
3 points on the plane 2x – 3y = 6
2 vectors + point => vector sum.
Move points around to show that a linear combination of two host vectors can reach any point on the plane.



Hence any two vectors and a point can define a plane. Equally any three points can define a plane.

Autograph file: 4b. planes.agg

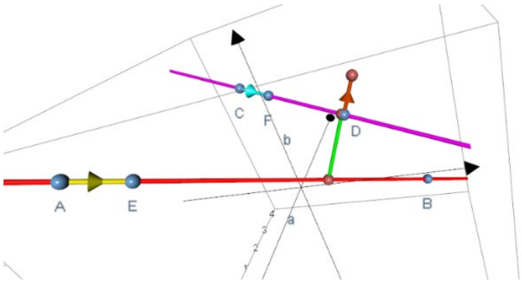
Draw and select two planes, draw the line of intersection. Put on a point, select the point and each plane to create two normal unit vectors. Hence the cross product.



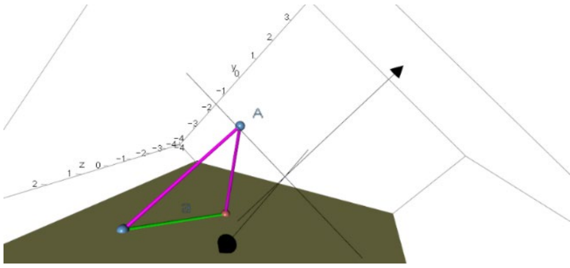
Autograph file: 4b. cross planes.agg

4c. CLOSEST POINT

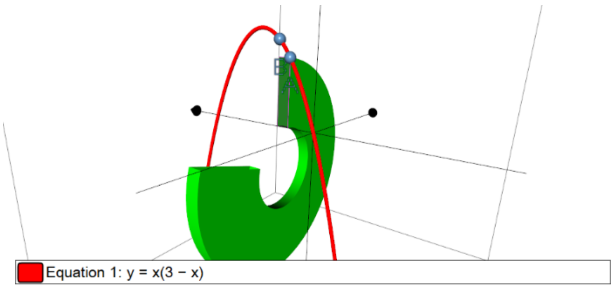
Shortest distance between two lines
Autograph file: 4c. shortest.agg

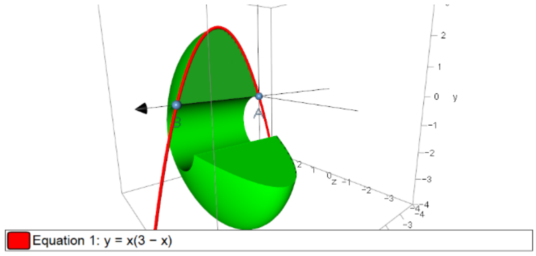


Closest point on a plane
Autograph file: 4c. closest.agg

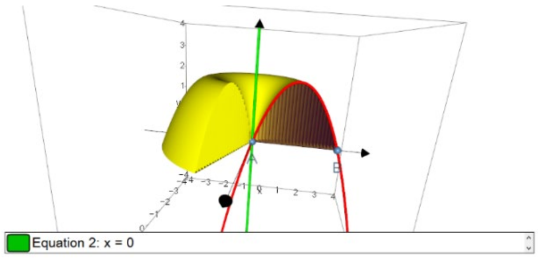


5a. Volume of revolution



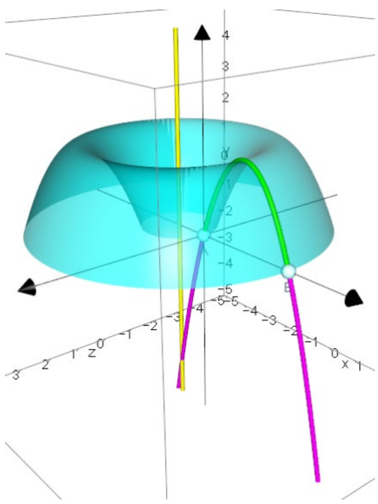


Autograph file: vol-element.agg
Autograph file: vol-y.agg
Autograph file: vol-x.agg



5b. Arc Length and Surface of revolution

Autograph file: 5b.surface.agg

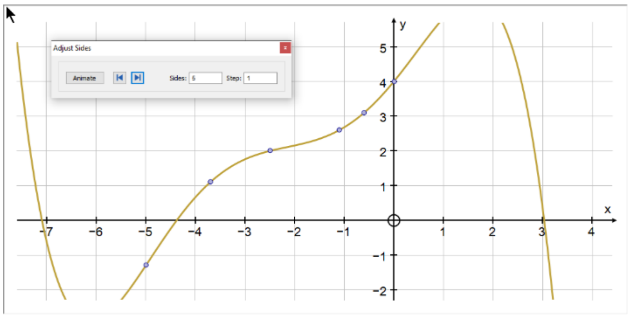


ADVANCED AUTOGRAPH

SESSION 12: Data and Probability Tools

1. FITTING MODELS TO DATA

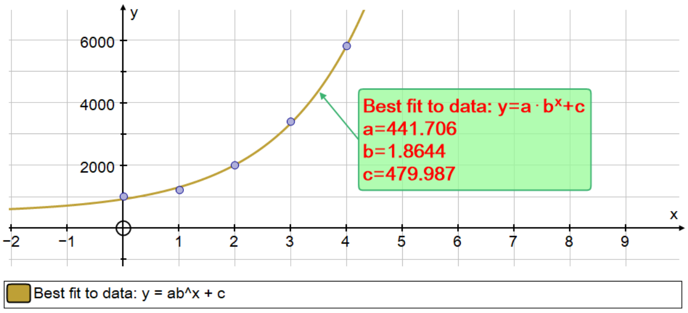
Place 6 points on screen, almost linear
Select all -> “Convert to data set”
Create -> “Best Fit polynomial”
Select this, and the “Animation controller”
Explore the order of polynomial
(max 5 for 6 points)



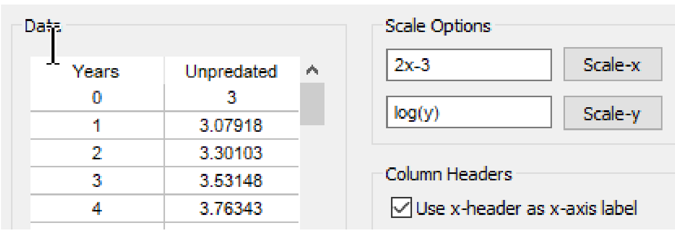
Create some data that is growing fast

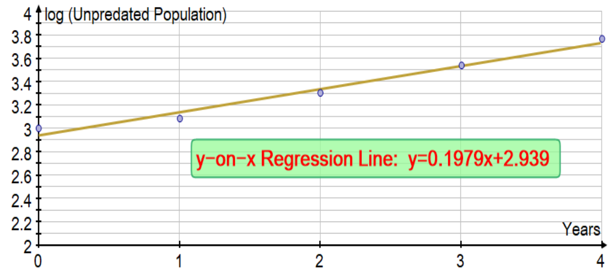
Choose a model: **y = a b^x + c**
Manipulate a, b, c using constant controller
Select the graph and the data to
finish it off: Create -> “Best Fit to Data”

Autograph file: 1a. best fit1.agg



Select/edit the data, and scale log(y):



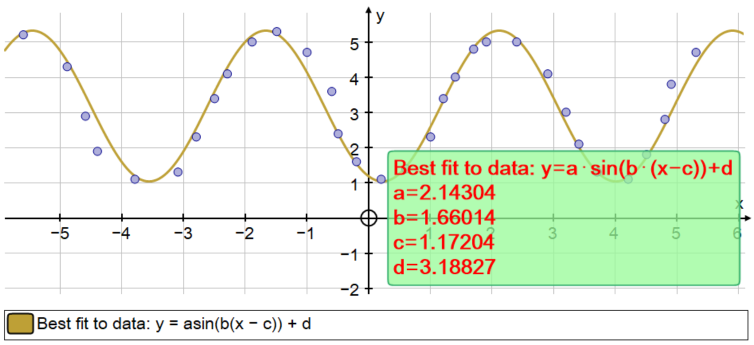


Autograph file: 1b. best fit1-log.agg

Create a few cycles of trig data
(eg tides)
Use the constant controller to
explore **y = asin(b(x-c)) + d**

and finally Create -> “Best Fit to Data”

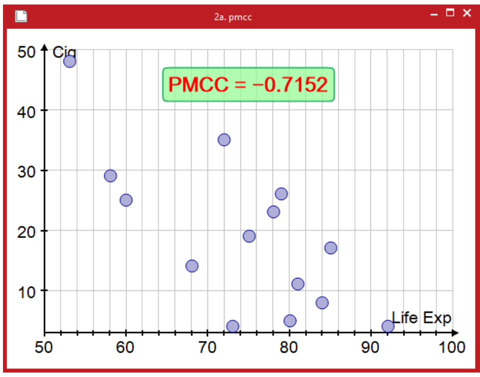
Autograph file: 1c. best fit2.agg

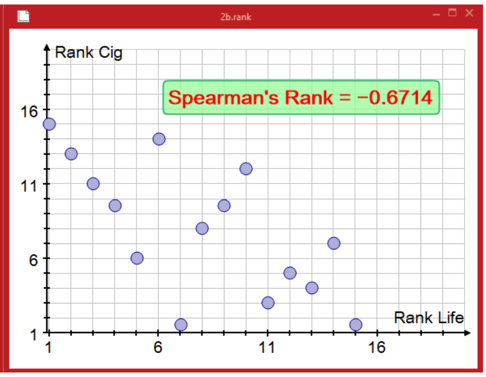


ADVANCED AUTOGRAPH

2. SPEARMAN' RANK/CORRELATION COEFFICIENT

Life Exp	Cig	Rank Life	Rank Cig
80	5	11	3
78	23	9	9.5
60	25	3	11
53	48	1	15
85	17	14	7
84	8	13	4
73	4	7	1.5
79	26	10	12
81	11	12	5
75	19	8	8
68	14	5	6
72	35	6	14
58	29	2	13
92	4	15	1.5
65	23	4	9.5

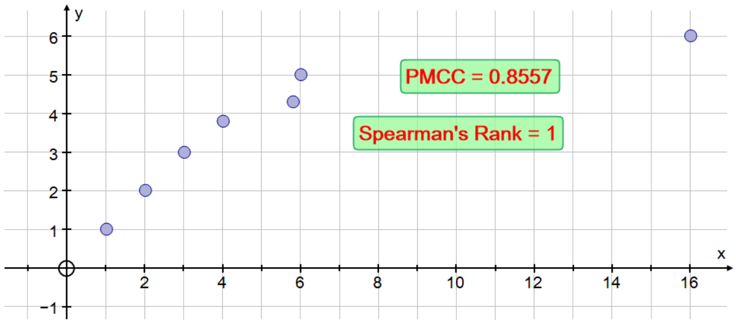




Data on Cigarettes smoking and Life Expectancy
Excel file: 2a. pmcc-rank.xlsx
Autograph files: 2a. pmcc.agg, 2b. rank.agg

With just 6 points, move (with CTRL) an individual point to illustrate the difference between PMCC and Ranking.

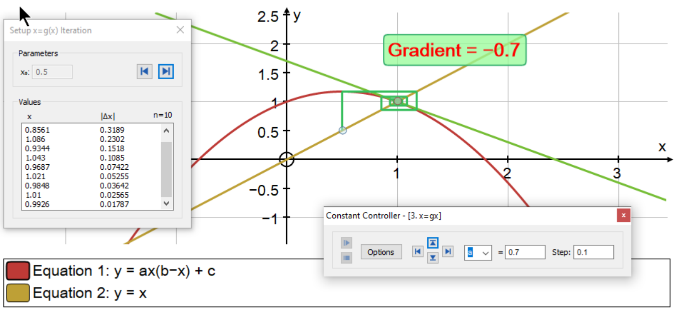
Autograph file: 2c. pmcc-rank.agg



3. NUMERICAL METHODS

x = g(x) iteration

Draw **y=x** and **y=ax(b-x)+c**
Draw a start point ON y=x. Select this point and the curve. Create -> x=g(x) Iteration.
A results dialogue opens so can control the progress of the iterations.
Find the intersection, and (double-click) to confirm that the point is associated with the graph. Draw the tangent.
Use the calculator to display the value of the gradient
Vary a, b or c and note that the iterations diverge when the gradient is numerically > 1.
Move the start point along the line y=x.

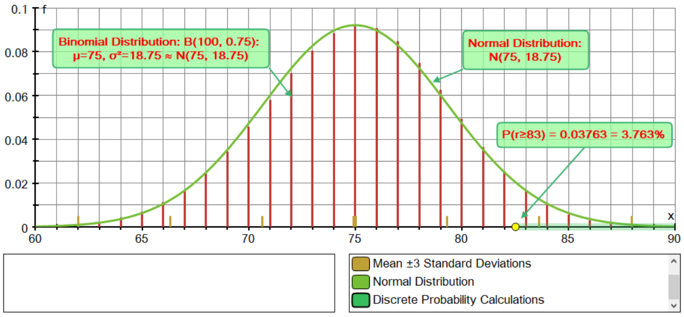


Autograph file: 3. x=gx.agg
Try a more complicated one: **y = (−x² + 4x − 1)^(1/3)** and **y = x**

4a. BINOMIAL PROBABILITY DISTRIBUTION

Binomial distribution B(100,0.75)
Mean and 3SDs
Normal approximation
Probability calculation
Table of statistics (in results box)

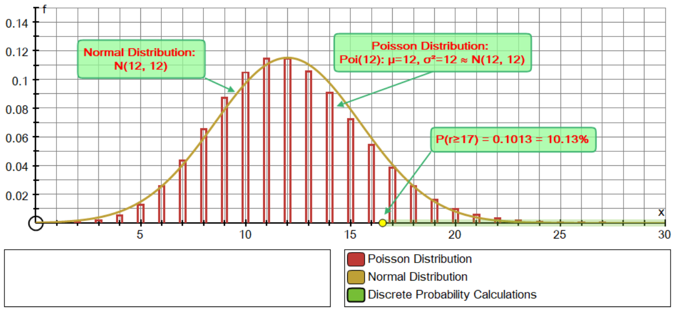
Autograph file: 4a. binomial.agg



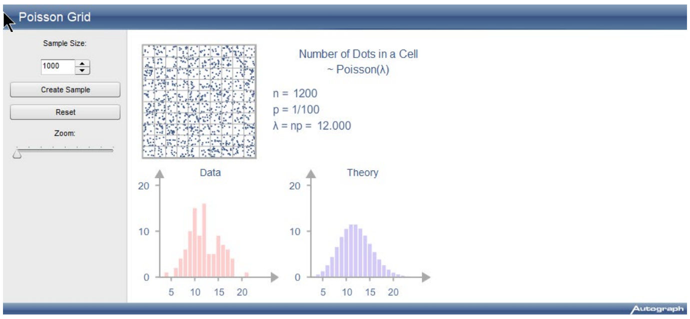
4b. BINOMIAL PROBABILITY DISTRIBUTION

Poisson distribution Poi(12)
Normal approximation
Probability calculation

Autograph file: 4b. poisson.agg

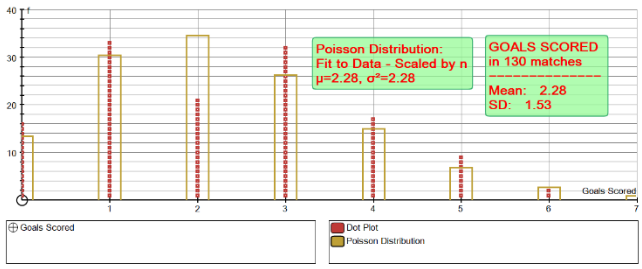


Poisson grid simulation
in File -> "New Extras Page"
A good simulation of the V2 rocket attacks on London at the end of the 2nd World War, which were fired randomly.



Simulating Premier League goals scored

	A	B	C	D	E
1	Soccer (misc Premiership matches, 2015-16)				
2	A	B	A+B	|A–B|	
3	1	3	4	2	
4	2	1	3	1	
5	2	1	3	1	
6	1	3	4	2	
7	1	0	1	1	
8	4	0	4	4	

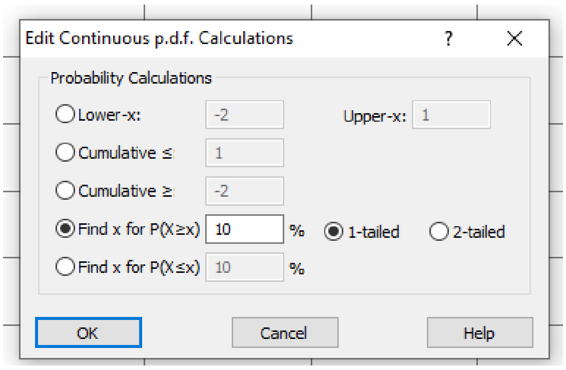


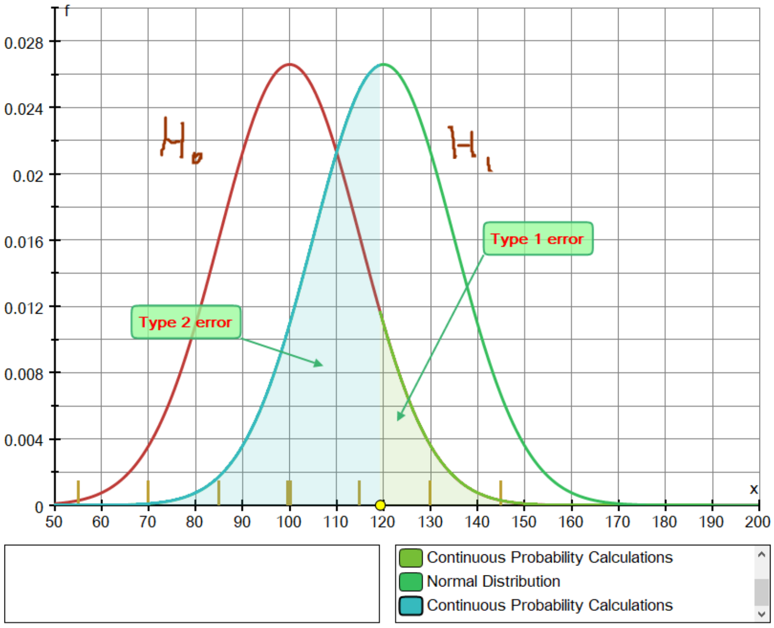
Copy the column "A+B" and enter raw data on a Statistics page in Autograph.
Plot a dot-plot, then enter a Poisson distribution, using "Fit to Data"

Autograph file: 4c. goals.agg

Excel file: 4c. SportsComparisons.xlsx

5. NORMAL DISTRIBUTION:
HYPOTHESIS TESTING





Autograph file: 5. normal.agg

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EXTENSION TOPICS

1. COMPLEX NUMBERS AND THE ARGAND DIAGRAM

2. POLAR PLOTTING

3. 2ND ORDER DIFFERENTIAL EQUATIONS/SHM

4. ARC LENGTH AND SURFACE OF REVOLUTION

5. CONSTRUCTION OF A CYCLOID

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