
A LEVEL: MEI MATHEMATICS TOPICS FOR AUTOGRAPH

Pre-release material: LARGE DATA SET

Topics in red require (for now) Autograph 5

PURE MATHEMATICS: ALGEBRA

a3 Solve $ax^2 + bx + c = 0$; discriminant $b^2 - 4ac$
a5 Simultaneous equations
a7 Inequalities eg $y - x > 1$; $y > ax^2 + bx + c$

PURE MATHEMATICS: FUNCTIONS

f4 f(x), g(x) then y = g(f(x))
f6 Graphs including modulus
C3 Quadratics: completing the square
C4 Graphing polynomials; repeated roots
C5 Turning points
C6 $y = a/x$ and $y = a/x^2$; asymptotes

PURE MATHEMATICS: GRAPHS

C7 y=af(x), y = f(x) + a, y = f(x + a), y = f(ax)
C8 Transformations of a graph
C9 Stationery points and point of inflexion

PURE MATHEMATICS: COORDINATE GEOMETRY

g1 Lines: // $m_1 = m_2$; $\perp$ $m_1m_2 = -1$
g2 Distance between 2 points; mid-point
g4 St line: $y - y_1 = m(x - x_1)$, $ax + by + c = 0$
g6 Intersection: 2 lines, line and curve, 2 curves
g9 Intersection: line and circle
g10 Circle: $(x - a)^2 + (y - b)^2 = r^2$; angle properties
g12 Parametric eqns: cartesian to parametric
g14 Parametric eqns: circle
g15 Gradient: dy/dx = (dy/dt)/(dx/dt)
g16 Kinematics and projectiles

PURE MATHEMATICS: TRIGONOMETRY

t1 Graphs of y = sinθ, cosθ, tanθ for any angle
t2 Symmetry, period, transformations
t5 $\tan\theta = \sin\theta/\cos\theta$
t6 $\sin^2\theta + \cos^2\theta = 1$
t7 Solving trig equations, eg $\sin\theta = 0.5$ [0°, 360°]
t9 arcsinx, arccosx, arctanx; $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$
t10 The radian
t11 Circle: arc length $s = r\theta$, sector $A = \frac{1}{2}r^2\theta$
t12 Small angles: $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$
t13 secθ, cosecθ, cotθ
t14 Graphs of sinθ, cosθ, tanθ, cosecθ, secθ, cotθ
t15 $\tan^2\theta + 1 = \sec^2\theta$, $\cot^2\theta + 1 = \text{cosec}^2\theta$
t16 Trig identities: sin(θ±ϕ), cos(θ±ϕ), tan(θ±ϕ)
t17 Double angles: sin2θ, cos2θ, tan2θ
t18 $a\cos\theta \pm b\sin\theta = r\sin(\theta \pm \alpha)$ and $r\cos(\theta \pm \alpha)$

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PURE: EXPONENTIALS AND LOGARITHMS

E1 y = a^x; x = a^y <=> y = logax
E7 Reduce exponential data to linear form
E8 $y = e^x$
E9 Gradient of $y = e^{kx}$ is ke^{kx}
E10 y = lnx and its inverse $y = e^x$
E11 Exponential growth and decay

PURE MATHEMATICS: CALCULUS

c1 Gradient at a point: gradient of tangent
c3 $dy/dx \approx \lim(\delta y/\delta x)$; $\lim(((f(x+h)-f(x))/h)$
c4 Sketch the gradient function
c5 Differentiating $y = kx^n$
c6 The second derivative; max and min
c8 The sign of dy/dx
c9 Equation of tangent and normal
c10 Differentiate: e^{kx}, a^{kx}, ln(x)
c11 Differentiate: sinkx, coskx, tankx
c12 Differentiate: Product rule; Quotient rule
c14 Differentiate: dy/dx = ((dy/du)×(du/dx))
c15 Differentiate: dy/dx = 1/((dx/dy))
c16 Differentiate: implicit functions eg $(x+y)^2 = 2x$
c19 Integrate: fundamental theorem of calculus
c20 Integrate: $y = kx^n$
c21 Integrate: constant; indefinite and definite
c23 Integrate: area under a curve
c24 Integrate: e^{kx}, 1/x, sinxkx
c25 Integrate: limit of a sum
c26 Integrate: area between two curves
c32 1st order differential eqns: separating vars.
c33 1st order differential eqns: links to kinematics

PURE MATHEMATICS: NUMERICAL METHODS

e3 $x=g(x)$: $x^3-x-4=0$ -> $x=(x+4)^{1/3}$
e4 Newton-Raphson method; failures
c34 Integration: trapezium rule; rectangles
c35 Integration: upper and lower bound

PURE MATHEMATICS: VECTORS

v1 2D vectors: modulus, unit vector, parallel
v2 2D vectors: add, subtract, multiply by a scalar
v3 2D vectors: magnitude and direction
v4 2D vectors: position vectors
v5 2D vectors: distance between two points
v6 2D vectors: vectors representing forces
v7 3D vectors

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STATISTICS: SAMPLING

p21 Population and sample; Sampling techniques

STATISTICS: DATA

D1 Discrete, continuous, ranked, grouped;
bar chart, dot plot, stem-and-leaf
box-and-whisker, frequency chart
D2 Histogram: frequency density
D3 Cumulative frequency diagram
D4 Describe: symmetrical, skewed
D6 Bivariate: scatter diagram; regression line
Extrapolation: outliers; correlation
D9 Time series
D10 Central tendency: median, mode, mean
D11 Measures of spread: range, quartiles, IQR
D12 Variance and standard deviation
D13 Outliers: mean ± 2SD; 1.5 × IQR

STATISTICS: PROBABILITY DISTRIBUTIONS

R3 Binomial: calculate probabilities; mean = np
R7 Discrete uniform distribution
R8 Normal Distribution: continuity correction
Normal Distribution: binomial approximation
R9 Normal Distribution: area -> probability
R10 Normal Distribution: $z = (x - \mu)/\sigma$
R11 Normal: mean ± σ = points of inflexion

STATISTICS: STATISTICAL HYPOTHESIS TESTING

H1 Null/alternative, 1, 2-tailed test
H6 Critical and acceptance regions
H7 Samples, n, from X -> $N(\mu, \sigma^2)$ -> $N(\mu, \sigma^2/n)$
H8 Test using Normal: Critical regions

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MECHANICS: KINEMATICS IN 1 DIMENSION

k4 Displacement, velocity, acceleration, time;
significance of gradient and area
k7 Const. Accel.: $s = ut+\frac{1}{2}at^2$, $v = u+at$, $v^2-u^2=2as$

MECHANICS: KINEMATICS IN 2 DIMENSIONS

k9 2D vectors: distance, velocity, acceleration
k11 Equation of path from vectors

MECHANICS: PROJECTILES

y1 Motion under gravity, using vectors
y2 Position, velocity, range, max height
y3 Initial velocity; Angle of projection
v5 Trajectory of a projectile
v6 Range on a uniform slope

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References to
MEI A LEVEL MATHEMATICS
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MEI FURTHER MATHEMATICS TOPICS FOR AUTOGRAPH

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CORE PURE: COMPLEX NUMBERS

j1 Real, imaginary, conjugate, modulus
argument, imaginary axis
j2 Solving any quadratic; conjugate pairs;
solving cubic/quartic equations
j4 Add, subtract, multiply, divide; Zero
j7 Modulus-argument form: $z = r(\cos\theta+i\sin\theta)$
j8 $z_1z_2=r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$
$z_1/z_2 = r_1/r_2(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2))$
j9 Sum, difference, product, quotient
j11 Sets of complex numbers as loci
Circles of the form $|z-a| = r$
Half lines of the form $\arg(z-a)=\theta$
Lines of the form $|z-a| = |z-b|$
j12 De Moivre's theorem: $z = e^{(i\theta)}= \cos\theta + i\sin\theta$
j17 nth roots: sum are zero

CORE PURE: MATRICES AND TRANSFORMATIONS

m4 2-D: transformations using matrices
reflect, rotate, enlarge, stretch, shear
3-D: transformations using matrices
reflection in x=0, y=0, z=0
rotation 90° about x, y or z axis
m7 Determinant of 2x2 and 3x3; singular
m8 Area scale factor
m9 Determinant of 3x3

CORE PURE: VECTORS AND 3-D SPACE

v1 Dot product; angle between 2 vectors
$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta = a_1b_1+a_2b_2+a_3b_3$
v2 Vector equation of a plane: $(\mathbf{r}-\mathbf{a}).\mathbf{n}=0$; $\mathbf{d}=-\mathbf{a}.\mathbf{n}$
Cartesian form of plane: $n_1x+n_2y+n_3z+d=0$
v3 Vector perpendicular to a plane
v4 3 distinct planes in 3-D
v5 3 linear simultaneous eqns; intersections
v6 Angle between two planes
v7 Vector product, perpendicular to two vectors
v8 Vector product, $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\mathbf{n}$
v9 Vector equation of a line in 2D and 3D
v10 Angle between two lines
v11 2 lines in 3D: intersection, shortest distance
v14 Intersection of a line and plane
v15 angle between a line and a plane
v16 Distance from a point to a line
v17 Distance from a point to a plane

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CORE PURE: SERIES

s3 Maclaurin: e^x, ln(1+x), sinx, cosx, $(1+x)^n$

CORE PURE: CALCULUS

c1 Definite integrals
c2 Volume of revolution about x- or y-axis
c3 Mean value of a fn: $1/(b-a)\int f(x)dx$, limits a-b
c4 Differentiate inverse trig functions

CORE PURE: POLAR COORDINATES

P1 Polar coordinates
P2 r = a(1+cosθ), r=acos2θ [r<0 broken line]
P3 Area enclosed by a polar curve $A = \frac{1}{2}\int r^2 d\theta$

CORE PURE: HYPERBOLIC FUNCTIONS

a3 sinhx, coshx, tanhx; $\cosh^2 x - \sinh^2 x = 1$
a5 differentiate and integrate hyperbolics
a6 inverse hyperbolic functions; log forms

CORE PURE: DIFFERENTIAL EQUATIONS

c7 1st order: general soln and particular integral
c8 Integrating factor: y' + P(x)y = Q(x)
c11 2nd order: y"+ay'+by=0 auxiliary equation
c12 Interpretation of the discriminant
c13 y"+ay'+by=f(x)
v11 SHM x"+cx=0, $x" = -\omega^2(x+k)$ -> $x = A\cos(\omega t - \phi)$
v12 Amplitude, T period = $(2\pi)/\omega$, $v^2 = \omega^2(A^2 - x^2)$
c16 Damped SHM
c17 Critical damping, roots of auxiliary equation
c18 Coupled 1st order linear, eg predator-prey

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STATISTICS: DISCRETE RANDOM VARIABLES

R7 Discrete uniform distribution
R10 Binomial distribution: $\mu=np$, $\sigma^2=npq$
R11 Poisson distribution: $\mu=\lambda$, $\sigma^2=\lambda$
R16 Geometric distribution: $\mu=1/p$, $\sigma^2=(1-p)/p^2$

STATISTICS: BIVARIATE DATA

b1 Scatter diagram; outliers; PMCC
b8 Spearman's Rank
b11 Equation of least squares regression
b12 Residuals; Two regression lines

STATISTICS: CONTINUOUS RANDOM VARIABLES

R19 Uniform and Normal distributions
R29 Normal distribution
R30 Central Limit Theorem

STATISTICS MAJOR: INFERENCE

I3 Distribution of the sample mean
I4 Central Limit Theorem
I7 Confidence Intervals

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NUMERICAL METHODS: SOLVING EQUATIONS

e1 Bisection method,
staircase and cobweb diagrams
e3 Fixed point iteration; Newton-Raphson

NUMERICAL METHODS: INTEGRATION

c3 Midpoint and trapezium rules;
Simpson's rule

EXTRA PURE: MULTIVARIABLE CALCULUS

c1 $z=f(x,y)$ is a surface, eg $z = xy^2 - 4x^2y + 20$

FURTHER PURE WITH TECHNOLOGY: INVESTIGATION OF CURVES

C1 Family of curves in cartesian,
polar and parametric forms
C8 Arc length
C9 Envelope of a family of curves

FURTHER PURE WITH TECHNOLOGY: EXPLORING DIFFERENTIAL EQUATIONS

c1 1st order Differential equations
c2 Tangent field
c7 Euler method or a modified Euler method
c9 Runge-Kutta method

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References to
MEI A LEVEL FURTHER MATHEMATICS
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