

INTERNATIONAL BACCALAUREATE  
DIPLOMA PROGRAMME 2008  
MATHEMATICS



In **BOLD** and **RED**: suitable for **AUTOGRAPH**

**AUTOGRAPH PAGE**

**IB MATHEMATICAL STUDIES SL (Standard Level)**

(first examined 2006)

**Studies TOPIC 1: INTRODUCTION TO THE G. D. CALCULATOR**

1.1 Arithmetic calculations; data; lists

**Studies TOPIC 2: NUMBER AND ALGEBRA**

- 2.1 Natural Numbers:  $\mathbb{N}$ ; Integers:  $\mathbb{Z}$ ; Rational numbers:  $\mathbb{Q}$ ; Real numbers:  $\mathbb{R}$
- 2.2 Approximation; sig. fig.
- 2.3 Standard form
- 2.4 SI units
- 2.5 Arithmetic sequences
- 2.6 Geometric sequences
- 2.7 **Two simultaneous equations; solving quadratic equations**



**Studies TOPIC 3: SETS, LOGIC AND PROBABILITY**

- 3.1 Set theory; prime numbers
- 3.2 Venn diagrams
- 3.3 Sample space
- 3.4 Symbolic logic
- 3.5 Compound statements
- 3.6 Truth tables
- 3.7 Logical equivalence
- 3.8 Probability
- 3.9 Venn diagrams; tree diagrams; cards, **2-dice**
- 3.10 Combined events/conditional probability

**Extras: "2-dice"**

**Studies TOPIC 4: FUNCTIONS**

- 4.1 **Functions: domain and range**
- 4.2 **Linear functions,  $y = mx + c$**
- 4.3 **Quadratic functions: vertex, symmetry  $x = -b/(2a)$**
- 4.4  **$y = a^x$ ,  $a^{kx}$ ,  $ka^{kx} + c$ ; exponential growth and decay**
- 4.5 **Trig (degrees):  $y = a \sin(bx) + c$ ;  $y = a \cos(bx) + c$**
- 4.7 **Graph sketching; rational graphs**
- 4.8 **Solving equations and intersections**



### Studies TOPIC 5: GEOMETRY AND TRIGONOMETRY

- 5.1 **Coordinate geometry; distance, mid points**
- 5.2 **Straight lines:  $y = mx + c$  and  $ax + by + d = 0$ ; perpendicular lines**
- 5.3 Right-angled trig
- 5.4 Sine and cosine rules
- 5.5 3D shapes: surface area and volume; New shapes from mid-points

**Autograph**



2D

### Studies TOPIC 6: STATISTICS

- 6.1 **Discrete, continuous**
- 6.2 **Frequency polygons**
- 6.3 **Grouped data, histogram (equal classes); stem and leaf diagrams**
- 6.4 **Cumulative Frequency, box plots, percentiles, quartiles**
- 6.5 **Mean, median, mode, percentile**
- 6.6 **Inter-quartile range, standard deviation; population and sample**
  
- 6.7 **Scatter diagrams, line of best fit; correlation**
- 6.8 **Regression line (y on x)**
- 6.9 Hypothesis testing; contingency tables. Chi-squared test



STATISTICS



STATISTICS



2D

### Studies TOPIC 7: INTRODUCTORY DIFFERENTIAL CALCULUS

- 7.1 **Gradient of chord PQ and  $P \Rightarrow Q$ ; tangent to a curve**
- 7.2 **Basic principles for  $ax^n$ :  $f'(x)$  and  $f''(x)$**
- 7.3 **Gradient of a curve; equation of tangent**
- 7.4 **Increasing and decreasing functions**
- 7.5 **Max and min; point of inflexion with zero gradient**



2D



2D

### Studies TOPIC 8: FINANCIAL MATHEMATICS

- 8.1 Currency conversions
- 8.2 Simple interest
- 8.3 Compound interest
- 8.4 Tables; inflation

### Studies PROJECT (20%)

**involving the collection of information or the generation of measurements, and the analysis and evaluation of the information or measurements.**



STATISTICS



2D

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# IB MATHEMATICS SL (Standard Level)

(first examined 2008)

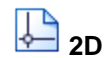


## SL TOPIC 1: ALGEBRA

- 1.1 APs, GPs; population growth
- 1.2 Exponents and logarithms; change of base
- 1.3 Binomial Theorem; Pascal's Triangle

## SL TOPIC 2: FUNCTIONS AND EQUATIONS

- 2.1 **Domain and range; Composite Functions  $f(g(x))$ .  
Inverse function**
- 2.2 **Graphing functions; vertical and horizontal asymptotes; roots**
- 2.3 **Transformation of graphs: translation, stretch, reflection in axes.  
Trig graphs. The inverse function; reflection in  $y = x$**
- 2.4 **Reciprocal function and  $y = 1/x$**
- 2.5 **The quadratic: axis of symmetry  $x = -b/a$ ; completing the square**
- 2.6 **The quadratic: roots; discriminant**
- 2.7  **$y = a^x$  and its inverse:  $y = \log_a x$**
- 2.8  **$y = e^x$  and  $y = \ln x$**



2D



2D



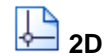
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2D

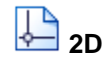
## SL TOPIC 3: CIRCULAR FUNCTIONS AND TRIGONOMETRY

- 3.1 **Radians**
- 3.2  **$\sin\theta$  and  $\cos\theta$  and the unit circle;  $\tan\theta = \sin\theta/\cos\theta$ ;  $\cos^2\theta + \sin^2\theta = 1$**
- 3.3 **Double angle formulae**
- 3.4 **Graphs of  $\sin x$ ,  $\cos x$ ,  $\tan x$ ;  $f(x) = a\sin(b(x + c)) + d$**
- 3.5 **Solving trig equations**
- 3.6 Sine and cosine rules



2D

Extras: "Trig"



2D

## SL TOPIC 4: MATRICES

- 4.1 Matrix : element, row, column, order
- 4.2 Matrix algebra
- 4.3 2D and 3D Determinants; 2D inverse
- 4.4 **Solving linear equations (2D and 3D)**



2D

## SL TOPIC 5: VECTORS

- 5.1 **2D and 3D vectors; distance between two points.  
Sum, difference; zero vector, negative vector;  
Scalar multiplication, magnitude, unit vector.**
- 5.2 **Scalar product; perpendicular vectors; angle between two vectors**
- 5.3 **Vector equation of a line; angle between two lines**
- 5.4 **Intersection of two lines**



2D



3D

## SL TOPIC 6: STATISTICS AND PROBABILITY

- 6.1 **Population and sample statistics; discrete and continuous**
- 6.2 **Box and whisker plots; grouped data; histogram (equal class intervals)**
- 6.3 **Mean, median, mode, quartiles; standard deviation**
- 6.4 **Cumulative frequency graph; percentiles.**
- 6.5 Probability
- 6.6 Probability: combined events
- 6.7 Conditional probability
- 6.8 Venn diagrams]
- 6.9 **Discrete probability distribution, eg:  $P(X=x) = 5/18, 6/18, 7/18$**   
Expected value for discrete data
- 6.10 **Binomial distribution; its mean**
- 6.11 **Normal Distribution; Standardisation; inverse calculations**

## SL TOPIC 7: CALCULUS

- 7.1 Ideas of limit and convergence  
**Basic principles; Derivative of  $x^n$ ,  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $e^x$ ,  $\ln x$**   
**Gradient; rate of change; Equations of tangents and normals**
- 7.2 **Chain rule, product and quotient rules; Second derivative**
- 7.3 **Local max and min; points of inflexion**
- 7.4 **Integration:  $x^n$ ,  $\sin x$ ,  $\cos x$ ,  $1/x$  and  $e^x$**
- 7.5 **Area under a curve; between two curves;**  
**Volume of revolution about x-axis**
- 7.6 **Displacement, velocity, acceleration, and time**  
**Area under v-t represents distance.**
- 7.7 **Horizontal and vertical asymptotes**  
**Second derivative: points of inflexion with non-zero gradient**

## SL PORTFOLIO (20%)

**Mathematical investigation**  
**Mathematical modelling**

**Autograph**

 STATISTICS

 STATISTICS

 STATISTICS

 STATISTICS

 2D

 2D

 3D

 2D

 STATISTICS

 2D,  3D

# IB MATHEMATICS HL (Higher Level)

(first examined 2008) [ITALIC = same as IB MATHEMATICS SL syllabus]

## HL TOPIC 1 - CORE: ALGEBRA


- 1.1 APs, GPs; population growth
- 1.2 Exponents and logarithms; change of base
- 1.3 Binomial Theorem; Pascal's Triangle
- 1.4 Proof by induction
- 1.5 **Complex numbers: Cartesian/polar form; modulus and argument; Argand diagram**
- 1.6 **Sum, product and quotient**
- 1.7 **De Moivre's Theorem; roots and powers**
- 1.8 **Complex conjugates**


 ARGAND [v.3.5]

 ARGAND [v.3.5]


## HL TOPIC 2 - CORE: FUNCTIONS AND EQUATIONS

- 2.1 *Domain and range; Composite Functions  $f(g(x))$ . Inverse function.*
- 2.2 *Graphing functions; vertical and horizontal asymptotes; roots*
- 2.3 *Transformation of graphs: translation, stretch, reflection in axes. Trig graphs. The inverse function; reflection in  $y = x$*   
 **$y = 1/f(x)$ ; graphs with absolute value, eg  $y = |f(x)|$ ,  $y f(|x|)$**
- 2.4 *Reciprocal function and  $y = 1/x$*
- 2.5 *The quadratic: axis of symmetry  $x = -b/a$ ; completing the square*
- 2.6 *The quadratic: roots; discriminant*
- 2.7  *$y = a^x$  and its inverse:  $y = \log_a x$*
- 2.8  *$y = e^x$  and  $y = \ln x$*
- 2.9 **Inequalities: in one variable;  $g(x) \geq f(x)$ , one linear and one quadratic**
- 2.10 **Roots of polynomial equations; repeated roots**

 2D

 2D

 2D

 2D

 2D

## HL TOPIC 3 – CORE: CIRCULAR FUNCTIONS AND TRIGONOMETRY

- 3.1 *Radians*
- 3.2  *$\sin\theta$  and  $\cos\theta$  and the unit circle;  $\tan\theta = \sin\theta/\cos\theta$ ;  $\cos^2\theta + \sin^2\theta = 1$ ;  $1 + \tan^2\theta = \sec^2\theta$ ;  $1 + \cot^2\theta = \csc^2\theta$ ;  $\sec\theta$ ,  $\csc\theta$ ,  $\cos\theta$ .*
- 3.3 *Double angle formulae;*  
**Compound angle Identities**
- 3.4 *Graphs:  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $\text{asin}(b(x + c)) + d$*   
**Inverse trig: arcsinx, arccosx, arctanx**
- 3.5 *Solving trig equations*
- 3.6 *Sine and cosine rules*

 2D  
Extras: "Trig"

 2D

 2D

## HL TOPIC 4 – CORE: MATRICES

- 4.1 Matrix : element, row, column, order
- 4.2 Matrix algebra
- 4.3 2D and 3D Determinants; 2D inverse
- 4.4 **Solving linear equations (2D and 3D)**

 2D,  3D

## HL TOPIC 5 – CORE: VECTORS

- 5.1 *2D and 3D vectors; distance between two points.  
Sum, difference; zero vector, negative vector;  
Scalar multiplication, magnitude, unit vector.*
- 5.2 *Scalar product; perpendicular vectors; angle between two vectors*
- 5.3 *Vector equation of a line; angle between two lines*  
**Parametric form:  $x = x_0 + \lambda l$ ,  $y = y_0 + \lambda m$ ,  $z = z_0 + \lambda n$**
- 5.4 *Intersecting and skew lines;  
Intersection of two lines*
- 5.5 **Vector product (cross product)**
- 5.6 **Vector equation of a plane; Equation of plane:  $ax + by + cz = d$**
- 5.7 **Intersections: line and plane, two planes, three planes.  
Angle between: line and plane, two planes**

Autograph



## HL TOPIC 6 - CORE: STATISTICS AND PROBABILITY

- 6.1 *Population and sample statistics; discrete and continuous*
- 6.2 *Box and whisker plots;  
grouped data; histogram (equal class intervals)*
- 6.3 *Mean, median, mode, quartiles; standard deviation*
- 6.4 *Cumulative frequency graph; percentiles.*
- 6.5 *Probability*
- 6.6 *Probability: combined events*
- 6.7 *Conditional probability*
- 6.8 *Venn diagrams*
- 6.9 *Discrete probability distribution, eg:  $P(X=x) = 5/18, 6/18, 7/18$*   
**Continuous probability density functions**  
*Expected value and Variance for discrete data*
- 6.10 *Binomial distribution; its mean and variance*  
**Poisson distribution: its mean and variance**
- 6.11 *Normal Distribution; Standardisation; inverse calculations*



## HL TOPIC 7 – CORE: CALCULUS

- 7.1 **Ideas of limit and convergence, eg  $\sin\theta/\theta$**   
*Basic principles; Derivative of  $x^n$ ,  $\sin x$ ,  $\cos x$ ,  $\tan x$ ,  $e^x$ ,  $\ln x$   
Gradient; rate of change; Equations of tangents and normals*
- 7.2 *Chain rule, product and quotient rules; Second derivative*
- 7.3 *Local max and min; points of inflexion*
- 7.4 *Integration:  $x^n$ ,  $\sin x$ ,  $\cos x$ ,  $1/x$  and  $e^x$*
- 7.5 *Area under a curve; between two curves;  
Volume of revolution about x-axis*
- 7.6 *Displacement, velocity, acceleration, and time  
Area under v-t represents distance.*
- 7.7 *Horizontal and vertical asymptotes  
Second derivative: points of inflexion with non-zero gradient*
- 7.8 **Implicit differentiation**
- 7.9 **Further Integration (substitution; parts)**
- 7.10 **First Order Differential Equations (variable separable)**



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## IB MATHEMATICS HL (Higher Level)

### OPTIONAL SYLLABUS CONTENT



#### HL TOPIC 8 – OPTION: STATISTICS AND PROBABILITY (CONTINUED)

- 8.1 Expectation algebra
- 8.2 **Cumulative distribution functions**  
**Discrete distributions: uniform, Bernoulli, binomial, negative binomial, Poisson, geometric, hypergeometric**  
**Continuous distributions: uniform, exponential, normal**
- 8.3 **Central limit theorem**
- 8.4 **Confidence intervals for the mean of a population**  
Confidence intervals for the proportion of successes in a population
- 8.5 **Null and alternative hypotheses: Type I and type II errors**  
**One-tailed and two-tailed test**
- 8.9  $\chi^2$  Goodness of fit test



#### HL TOPIC 9 – OPTION: SETS, RELATIONS AND GROUPS

- 9.1 Sets; De Morgan's Laws
- 9.2 Ordered pairs
- 9.3 Functions and inverse functions
- 9.4 Binary operations
- 9.5 Associative, distributive, commutative
- 9.6 Identity element; inverse
- 9.7 Axioms of a group
- 9.8 Groups
- 9.9 Finite and infinite groups
- 9.10 Cyclic groups
- 9.11 Subgroups, Lagrange theorems
- 9.12 Isomorphism of groups

#### HL TOPIC 10 – OPTION: SERIES AND DIFFERENTIAL EQUATIONS

- 10.1 Infinite sequences
- 10.2 Convergence
- 10.3 Convergent series
- 10.4 Power series
- 10.5 Taylor polynomials  
**Maclaurin series:  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\arctan x$ ,  $\ln(1+x)$ ,  $(1+x)^p$**   
**Limits of the form  $f(x)/g(x)$**   
L'Hôpital's Rule and/or the Taylor series
- 10.6 **First order differential equations - slope fields**  
 **$y' = f(x,y)$ : Numerical solution, Euler's Method**  
**Homogeneous DEs,  $y' = f(y/x)$**   
 **$y' + P(x)y = Q(x)$  using Integrating factor**

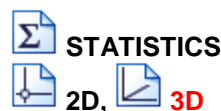


**HL TOPIC 11 – OPTION: DISCRETE MATHEMATICS**

- 11.1 Division and Euclidean algorithms
- 11.2 Integers in different bases
- 11.3 Linear diophantine equations
- 11.4 Modular arithmetic
- 11.5 Fermat's little theorem
- 11.6 Graphs
- 11.7 Trails and circuits
- 11.8 Adjacency matrix
- 11.9 Graph algorithms
- 11.10 Travelling Salesman

**HL PORTFOLIO (20%)**

**Mathematical investigation**  
**Mathematical modeling**



**IB FURTHER MATHEMATICS SL (Standard Level)**

**FM Topic 1 GEOMETRY**

- 1.1 **Triangles; Nine-point Circle; Euler Line**
- 1.2 Euclid's theorem for right-angled triangle  
Proportional division of a line; harmonic ratio
- 1.3 **Circle Geometry; equation of a circle; cyclic quadrilaterals**  
Intersecting Chords theorem
- 1.4 Apollonius' circle theorem; Stewart's theorem; Menelaus' theorem;  
Ceva's theorem; Simpson's line; Ptolemy's th.; angle bisector theorem

**Loci of straight lines and circles** [not conic sections]



- FM Topic 2 = HL TOPIC 8 – OPTION: STATISTICS AND PROBABILITY**
- FM Topic 3 = HL TOPIC 9 – OPTION: SETS, RELATIONS AND GROUPS**
- FM Topic 4 = HL TOPIC 10 – OPTION: SERIES AND DIFF. EQUATIONS**
- FM Topic 5 = HL TOPIC 11 – OPTION: DISCRETE MATHEMATICS**

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