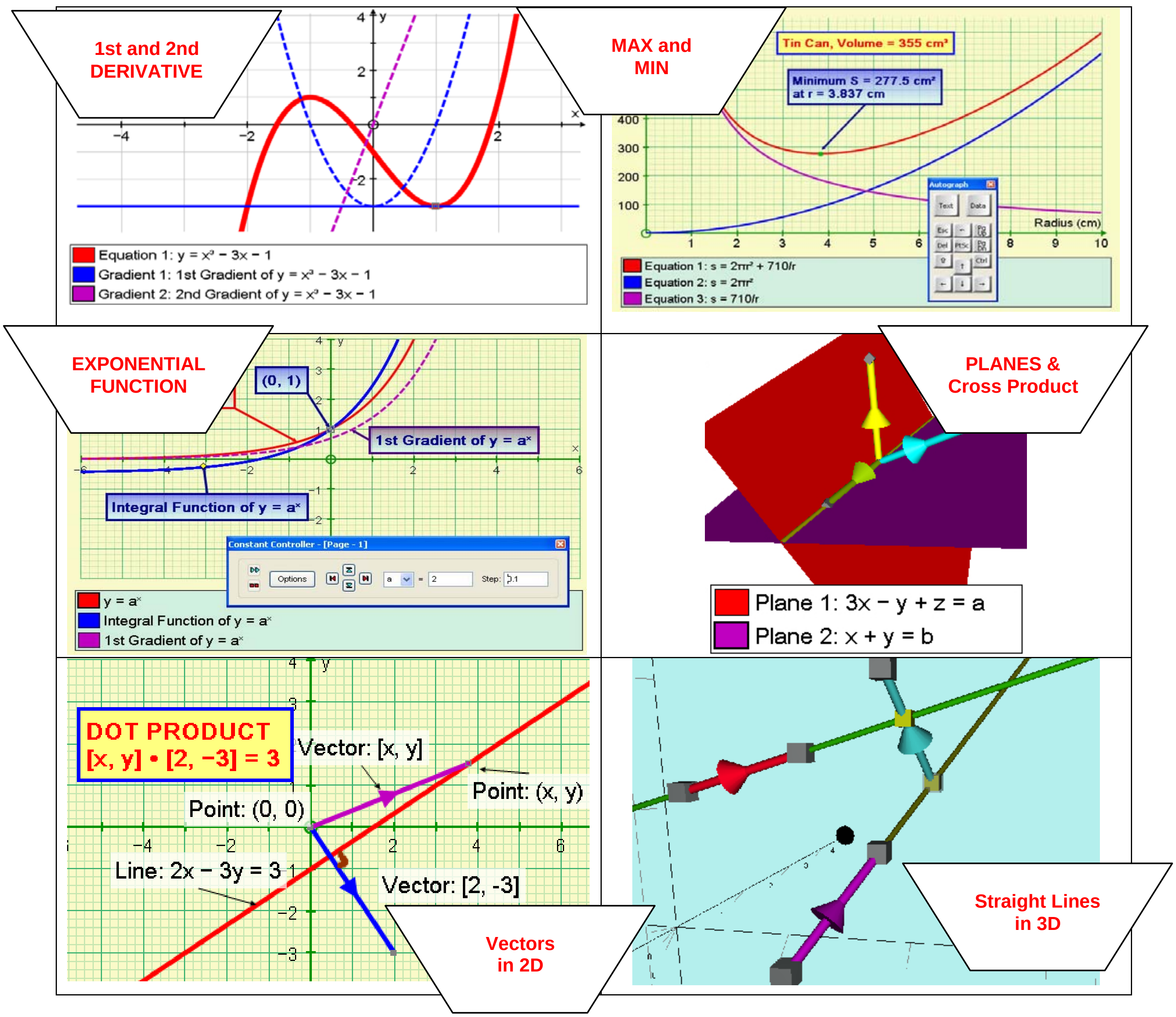
Autograph version 3

and Calculus and Vectors

Autograph is spectacular dynamic software from the UK that allows teachers to visualise many of the mathematical topics that occur in the Ontario Grade 12 CALCULUS and VECTORS course [MCV4U].



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Ministry of Education The Ontario Mathematics Curriculum [2007]

CALCULUS AND VECTORS

Grade 12 - University Preparation MCV4U

Edited to show Autograph associations (in red) [*]

==	AUTOGRAPH PAGE ==============
A. RATE OF CHANGE	
1. INSTANTANEOUS RATE OF CHANGE AT A POINT	
1.1 examples of real-world applications of rates of change	
1.2 connections between the average rate of change of a function and the slope of the corresponding secant, and between the instantaneous rate of change at a point and the slope of the tangent at that point	2D
1.3 make connections between an approximate value of the instantaneous rate of change at a given point on the graph of a smooth function and average rates of change over intervals containing the point	2D
1.4 recognize graphical and numerical examples of limits	2D
1.5 make connections between the average rate of change and the value of the expression (f(a + h)–f (a))/h, and between the instantaneous rate of change of the function at x = a and the value of the limit	2D
1.6 compare the calculation of instantaneous rates of change at a point (a, f(a)) for polynomial functions, with and without simplifying the expression before substituting values of h that approach zero	2D
2. THE CONCEPT OF THE DERIVATIVE FUNCTION	
2.1 determine numerically and graphically the intervals over which the instantaneous rate of change is positive, negative, or zero for a function that is smooth over these intervals	2D
2.2 generate a table of values showing the instantaneous rate of change of a polynomial function, f(x), for various values of x, graph the ordered pairs, recognize that the graph represents a function called the derivative	2D
2.3 determine the derivatives of polynomial functions by simplifying the algebraic expression (f(x+h)–f(x))/h, and then taking the limit of the simplified expression as h approaches zero.	2D
2.4 determine the graph of the derivative f ’(x) of a given sinusoidal function; verify graphically that when f(x) = sin x, f ’(x) = cos x, and when f(x) = cos x, f’(x) = – sin x	2D
2.5 determine the graph of the derivative f ’(x) or dy/dx of a given exponential function. Verify that when f(x) = a^x, f ’(x) = kf(x)], and make connections between the graphs of f(x) and f ’(x) or y and dy/dx.	2D

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2.6 determine the exponential function f(x) = a^x (a > 0, a ≠ 1) for which f'(x) = f(x); identify the number 'e' to be the value of a for which f '(x) = f(x)	2D
2.7 recognize that the natural logarithmic function f(x) = lnx is the inverse of the exponential function f(x) = e^x , and make connections between f(x) = ln x and f(x) = e. their graphs are reflections of each other in the line y = x	2D
2.8 verify that the derivative of the exponential function f(x) = a^x is f '(x) = a^x ln a, for various values of a	2D
3. THE PROPERTIES OF DERIVATIVES	
3.1 the power rule for f(x) = x^n , where n is a natural number	2D
3.2 verify the constant, constant multiple, sum, and difference rules graphically and numerically	
3.3 determine algebraically the derivatives of polynomial functions	
3.4 verify that the power rule applies to functions of the form f(x) = x^n , where n is a rational number, and verify algebraically the chain rule using monomial functions and the product rule using polynomial functions	
3.5 solve problems, using the product and chain rules, involving the derivatives of polynomial functions, sinusoidal functions, exponential functions, rational functions	
B. DERIVATIVES AND THEIR APPLICATIONS	
1. GRAPHS and EQUATIONS OF FUNCTIONS AND THEIR DERIVATIVES	
1.1 sketch the graph of a derivative function, given the graph of a function that is continuous over an interval, and recognize points of inflection of the given function	2D
1.2 recognize the second derivative as the rate of change of the rate of change, and sketch the graphs of the first and second derivatives, given the graph of a smooth function	2D
1.3 determine algebraically the equation of the second derivative f''(x) of a polynomial or simple rational function f(x)	
1.4 describe key features of a polynomial function, given information about its first and/or second derivatives; sketch two or more possible graphs of the function that are consistent with the given information, and explain why an infinite number of graphs is possible	2D
1.5 sketch the graph of a polynomial function, given its equation, by using a variety of strategies to determine its key features (e.g., increasing or decreasing intervals, intercepts, local maxima and minima, points of inflection, intervals of concavity), and verify using technology	2D

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2. SOLVING PROBLEMS USING MATHEMATICAL MODELS + DERIVATIVES	
2.1 make connections between the concept of motion (i.e., displacement, velocity, acceleration) and the concept of the derivative in a variety of ways (e.g., verbally, numerically, graphically, algebraically)	
2.2 make connections between the graphical or algebraic representations of derivatives and real-world applications (e.g., population and rates of population change, prices and inflation rates, volume and rates of flow, height and growth rates)	2D
2.3 solve problems, using the derivative, that involve instantaneous rates of change, including problems arising from real-world applications	
2.4 solve optimization problems involving polynomial, simple rational, and exponential functions drawn from a variety of applications, including those arising from real-world situations	
2.5 solve problems arising from real-world applications by applying a mathematical model and the concepts and procedures associated with the derivative to determine mathematical results, and interpret and communicate the results	
C. GEOMETRY AND ALGEBRA OF VECTORS	
1. REPRESENTING VECTORS GEOMETRICALLY AND ALGEBRAICALLY	
1.1 recognize a vector as a quantity with both magnitude and direction, and identify, gather, and interpret information about real-world applications of vectors	2D
1.2 represent a vector in two-space geometrically as a directed line segment, with directions expressed in different ways (e.g., 320º; N 40º W), and algebraically (e.g., using Cartesian coordinates; using polar coordinates), and recognize vectors with the same magnitude and direction but different positions as equal vectors	2D
1.3 determine, using trigonometric relationships the Cartesian representation of a vector in 2D space given as a directed line segment, or the representation as a directed line segment of a vector in two-space given in Cartesian form	2D
1.4 recognize that points and vectors in 3D space can both be represented using Cartesian coordinates, and determine the distance between two points and the magnitude of a vector using their Cartesian representations	3D

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2. OPERATING WITH VECTORS	
2.1 perform the operations of addition, subtraction, and scalar multiplication on vectors represented as directed line segments in 2D space, and on vectors represented in Cartesian form in 2D space and 3D space	2D, 3D
2.2 determine some properties (e.g., commutative, associative, and distributive properties) of the operations of addition, subtraction, and scalar multiplication of vectors	2D, 3D
2.3 solve problems involving the addition, subtraction, and scalar multiplication of vectors, including problems arising from real-world applications	
2.4 perform the operation of dot product on two vectors represented as directed line segments and in Cartesian form in 2D space and 3D space, and describe applications of the dot product (e.g., determining the angle between two vectors; determining the projection of one vector onto another)	2D, 3D
2.5 determine properties of the dot product (e.g., investigate whether it is commutative, distributive, or associative; investigate the dot product of a vector with itself and the dot product of orthogonal vectors). eg the dot product of the vectors [1, 0, 1] and [0, 1, – 1] and the dot product of scalar multiples of these vectors.	3D
2.6 perform the operation of cross product on two vectors represented in Cartesian form in 3D; determine the magnitude of the cross product and describe applications of the cross product	3D
2.7 determine properties of the cross product	
2.8 solve problems involving dot product and cross product, e.g., determining projections, the area of a parallelogram, the volume of a parallelepiped;	3D
3. DESCRIBING LINES AND PLANES USING LINEAR EQUATIONS	
3.1 the solution points (x, y) in 2D space of a single linear equation in two variables form a line and that the solution points (x, y) in 2D space of a system of two linear equations in two variables determine the point of intersection of two lines, if the lines are not coincident or parallel	2D
3.2 determine with and without technology, that the solution points (x, y, z) in 3D space of a single linear equation in three variables form a plane and that the solution points (x, y, z) in 3D space of a system of two linear equations in three variables form the line of intersection of two planes, if the planes are not coincident or parallel	3D
3.3 determine different geometric configurations of combinations of up to three lines and/or planes in three-space (e.g., two skew lines, three parallel planes, two intersecting planes, an intersecting line and plane)	3D

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4. LINES +PLANES USING SCALAR, VECTOR, AND PARAMETRIC EQUATIONS	
4.1 recognize a scalar equation for a line in 2D space to be an equation of the form Ax + By + C = 0, represent a line in 2D space using a vector equation and parametric equations, and make connections between a scalar equation, a vector equation, and parametric equations of a line in 2D space	2D
4.2 recognize that a line in 3D space cannot be represented by a scalar equation, and represent a line in three-space using the scalar equations of two intersecting planes and using vector and parametric equations	3D
4.3 recognize a normal to a plane geometrically (i.e., as a vector perpendicular to the plane) and algebraically, and determine, through investigation, some geometric properties of the plane (e.g., the direction of any normal to a plane is constant; all scalar multiples of a normal to a plane are also normals to that plane; three non-collinear points determine a plane; the resultant, or sum, of any two vectors in a plane also lies in the plane)	3D
4.4 recognize a scalar equation for a plane in 3D space to be an equation of the form Ax + By + Cz + D = 0 whose solution points make up the plane, determine the intersection of three planes represented using scalar equations by solving a system of three linear equations in three unknowns algebraically (e.g., by using elimination or substitution), and make connections between the algebraic solution and the geometric configuration of the three planes	3D
4.5 determine, using properties of a plane, the scalar, vector, and parametric equations of a plane	3D
4.6 determine the equation of a plane in its scalar, vector, or parametric form, given another of these forms	3D
4.7 solve problems relating to lines and planes in three-space that are represented in a variety of ways (e.g., scalar, vector, parametric equations) and involving distances (e.g., between a point and a plane; between two skew lines) or intersections (e.g., of two lines, of a line and a plane), and interpret the result geometrically	3D

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[*] **The Ontario Curriculum, Grades 11 and 12**
The full document, Revised 2007 is at:
www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf

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