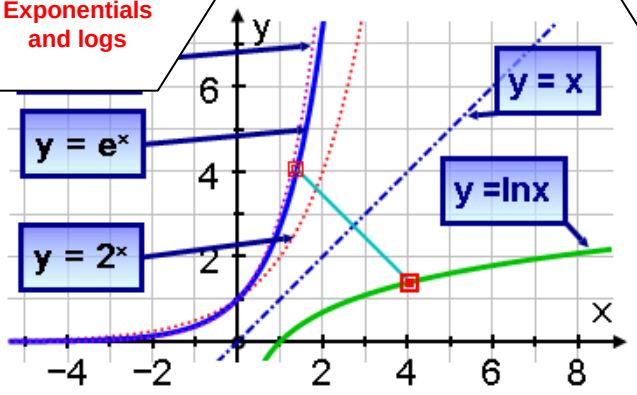
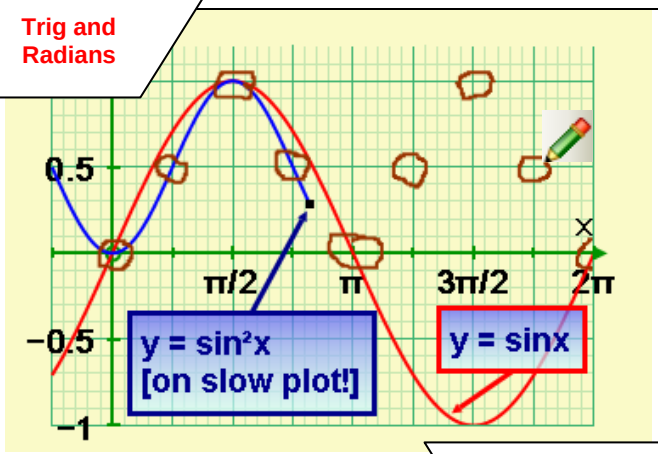
Autograph version 3 and Advanced Functions

Autograph is spectacular dynamic software from the UK that allows teachers to visualise many of the mathematical topics that occur in the Ontario Grade 12 ADVANCED FUNCTIONS course [MHF4U].

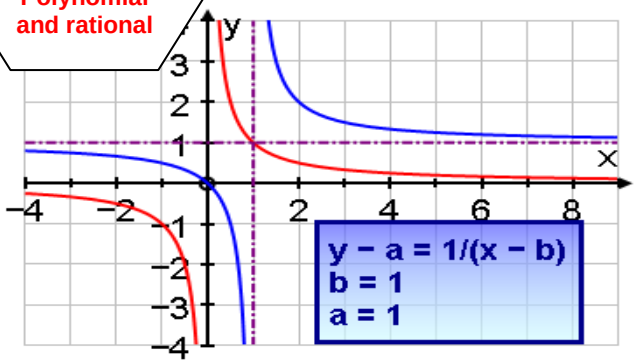
Exponentials and logs



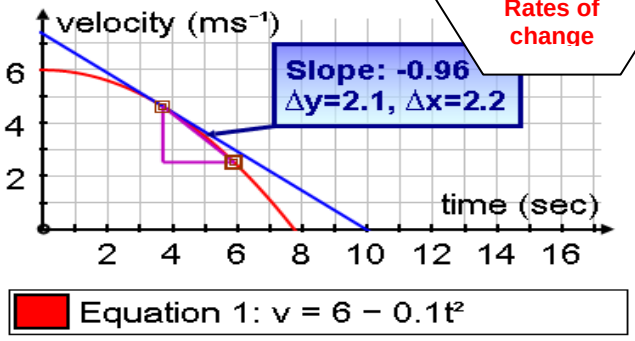
Trig and Radians



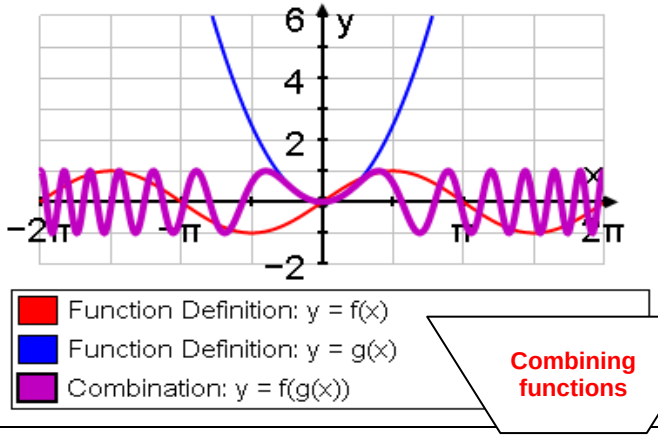
Polynomial and rational



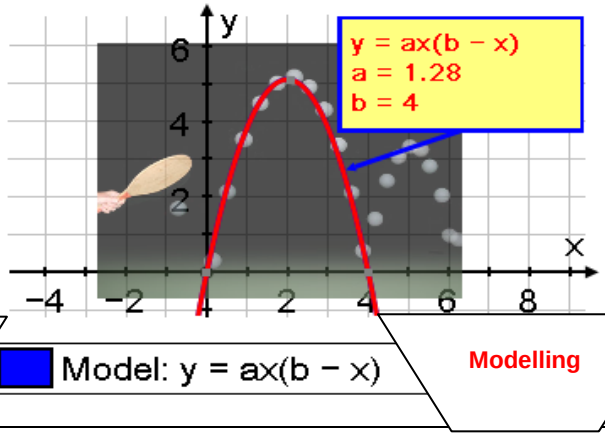
Rates of change



Combining functions



Modelling



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Ministry of Education The Ontario Mathematics Curriculum [2007]

ADVANCED FUNCTIONS
Grade 12 - University Preparation MHF4U

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AUTOGRAPH PAGE

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A. EXPONENTIAL AND LOGARITHMIC FUNCTIONS

1. EVALUATING LOGARITHMIC EXPRESSIONS

1.1 the logarithm of a number to a given base as the exponent to which the base must be raised to get the number, recognize the operation of finding the logarithm to be the inverse operation of exponentiation
1.2 the approximate logarithm of a number to any base, incl. base 10
1.3 connections between related logarithmic and exponential equations
1.4 make connections between the laws of exponents and the laws of logarithms [log x + log y = log (xy)]

2. CONNECTING GRAPHS AND EQUATIONS OF LOGARITHMIC FUNCTIONS

2.1 key features of the graphs of logarithmic functions, and make connections between the algebraic and graphical representations

 2D

2.2 the relationship between an exponential function and the corresponding logarithmic function; the graph of a logarithmic function is the reflection of the graph of the corresponding exponential function in the line y = x
2.3 the roles of the parameters d and c in functions of the form y = log (x – d) + c and the roles of the parameters a and k in functions of the form y = alog (kx)

 2D

2.4 pose problems based on real-world applications of exponential and logarithmic functions

3. SOLVING EXPONENTIAL AND LOGARITHMIC EQUATIONS

3.1 equivalent algebraic expressions involving logarithms and exponents
3.2 solve exponential equations in one variable by determining a common base
3.3 solve simple logarithmic equations in one variable algebraically
3.4 solve problems involving exponential and logarithmic equations algebraically

ADVANCED FUNCTIONS [Grade 12]

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B. TRIGONOMETRIC FUNCTIONS

1. UNDERSTANDING AND APPLYING RADIAN MEASURE

1.1 the radian as an alternative unit to the degree; angle as the length of the arc that subtends this angle at the centre of a unit circle
1.2 radian measure in terms of π and as a rational number
1.3 the primary trigonometric ratios and the reciprocal trigonometric ratios of angles expressed in radian measure
1.4 the exact values of the primary trigonometric ratios and the reciprocal trigonometric ratios for the special angles

2. CONNECTING GRAPHS AND EQUATIONS OF TRIG FUNCTIONS

2.1 sketch the graphs of f(x) = sin x and f(x) = cos x for angles in radians
2.2 connections between the tangent ratio and the tangent function
2.3 graphs of cosecant, secant and cotangent for angles in radians
2.4 the amplitude, period, and phase shift of sinusoidal functions in the form f(x) = a sin (k(x – d)) + c or f(x) = a cos(k(x – d)) + c, with angles in radians
2.5 sketch graphs of y = a sin (k(x – d)) + c and y = a cos(k(x – d)) + c by applying transformations to the graphs of f(x) = sin x and f(x) = cos x with angles expressed in radians
2.6 represent a sinusoidal function with an equation, given its graph or its properties, with angles expressed in radians
2.7 pose problems based on applications involving a trigonometric function with domain expressed in radians

 2D

 2D

 2D

3. SOLVING TRIGONOMETRIC EQUATIONS

3.1 recognize equivalent trigonometric expressions [e.g., sin x and cos (π/2 – x) are equivalent]
3.2 explore the algebraic development of the compound angle formulas
3.3 prove trigonometric identities through the application of reasoning skills, using a variety of relationships (e.g., tanx = sinx/cosx; $\sin^2 x + \cos^2 x = 1$; reciprocal identities; compound angle formulas
3.4 solve linear and quadratic trigonometric equations, for the domain of real values from 0 to 2π, and solve related problems

 2D

 2D

 2D

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C. POLYNOMIAL AND RATIONAL FUNCTIONS

1. CONNECTING GRAPHS AND EQUATIONS OF POLYNOMIAL FUNCTIONS

1.1 recognize a polynomial expression

1.2 investigate the effect of the degree of a polynomial function on the shape of its graph and the maximum number of x-intercepts —  2D

1.3 describe key features of the graphs of polynomial functions

1.4 distinguish polynomial functions from sinusoidal and exponential functions —  2D

1.5 make connections between a polynomial function given in factored form and the x-intercepts of its graph, and sketch the graph of a polynomial function given in factored form using its key features —  2D

1.6 the roles of the parameters a, k, d, and c in functions of the form $y = af(k(x – d)) + c$, and describe these roles in terms of transformations on the graphs of $f(x) = x^3$ and $f(x) = x^4$

1.7 determine an equation of a polynomial function that satisfies a given set of conditions —  2D

1.8 the equation of the family of polynomial functions with a given set of zeros and of the member that passes through another given point

1.9 determine, through investigation, and compare the properties of even and odd polynomial functions —  2D

2. CONNECTING GRAPHS AND EQUATIONS OF RATIONAL FUNCTIONS

2.1 key features of the graphs of rational functions that are the reciprocals of linear and quadratic functions, and make connections between the algebraic and graphical representations of these rational functions —  2D

2.2 key features of the graphs of rational functions that have linear expressions in the numerator and denominator —  2D

2.3 sketch the graph of a simple rational function using its key features, given the algebraic representation of the function

3. SOLVING POLYNOMIAL AND RATIONAL EQUATIONS

3.1 make connections between the polynomial function f(x), the divisor x – a, the remainder from the division, and f(a) to verify the remainder theorem and the factor theorem

3.2 factor polynomial expressions in one variable, of degree no higher than four, by selecting and applying strategies

3.3 the connection between the real roots of a polynomial equation and the x-intercepts of the graph of the corresponding polynomial function, and describe this connection —  2D

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3.4 solve polynomial equations in one variable, of degree no higher than four by selecting and applying strategies	
3.5 the connection between the real roots of a rational equation and the x-intercepts of the graph of the corresponding rational function	 2D
3.6 solve simple rational equations in one variable algebraically, and verify solutions using technology	
3.7 solve problems involving applications of polynomial and simple rational functions and equations	
4. SOLVING INEQUALITIES	
4.1 explain the difference between the solution to an equation in one variable and the solution to an inequality in one variable	
4.2 determine solutions to polynomial inequalities in one variable, and to simple rational inequalities in one variable by graphing	 2D
4.3 solve linear inequalities and factorable polynomial inequalities in one variable (e.g., $x^3 + x^2 > 0$) in a variety of ways	 2D
D. CHARACTERISTICS OF FUNCTIONS	
1. UNDERSTANDING RATES OF CHANGE	
1.1 gather, interpret, and describe information about real-world applications of rates of change	
1.2 distinguish situations in which the rate of change is zero, constant, or changing by examining applications, including those arising from real-world situations	 2D
1.3 sketch a graph that represents a relationship involving rate of change, as described in words, and verify with technology	 2D
1.4 calculate and interpret average rates of change of functions arising from real-world applications	
1.5 recognize examples of instantaneous rates of change arising from real-world situations, and make connections between instantaneous rates of change and average rates of change	 2D
1.6 approximate instantaneous rates of change arising from real-world applications by using average rates of change	
1.7 make connections between the slope of a secant on the graph of a function and the average rate of change of the function over an interval, and between the slope of the tangent to a point on the graph	 2D
1.8 the approximate slope of the tangent to a given point on the graph of a function by using the slopes of secants through the given point	 2D
1.9 solve problems involving average and instantaneous rates of change	

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2. COMBINING FUNCTIONS	
2.1 key features of the graphs of functions created by adding, subtracting, multiplying, or dividing functions	
2.2 recognize real-world applications of combinations of functions	
2.3 some properties (i.e., odd, even, or neither; increasing/decreasing behaviours) of functions formed by adding, subtracting, multiplying, and dividing general functions	2D
2.4 the composition of two functions [i.e., f(g(x))] numerically and graphically, with technology, for functions represented in a variety of ways	2D
2.5 determine algebraically the composition of two functions [i.e., f(g(x))], verify that f(g(x)) is not always equal to g(f(x))	
2.6 solve problems involving the composition of two functions, including problems arising from real-world applications	
2.7 the property that the composition of a function and its inverse function maps a number onto itself	2D
2.8 connections between transformations (i.e., vertical and horizontal translations; reflections in the axes; vertical and horizontal stretches and compressions to and from the x- and y-axes) of simple functions f(x)	2D
3. USING FUNCTION MODELS TO SOLVE PROBLEMS	
3.1 compare the characteristics of various functions (i.e., polynomial, rational, trigonometric, exponential, logarithmic)	2D
3.2 solve graphically and numerically equations and inequalities whose solutions are not accessible by standard algebraic techniques	2D
3.3 solve problems, including problems arising from real-world applications, by reasoning with functions and by applying concepts and procedures involving functions	

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[*] **The Ontario Curriculum, Grades 11 and 12**
The full document, Revised 2007 is at:
www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf

Oundle
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