

## The solutions of $x^6 = x^3$ using *Phantom Graphs*

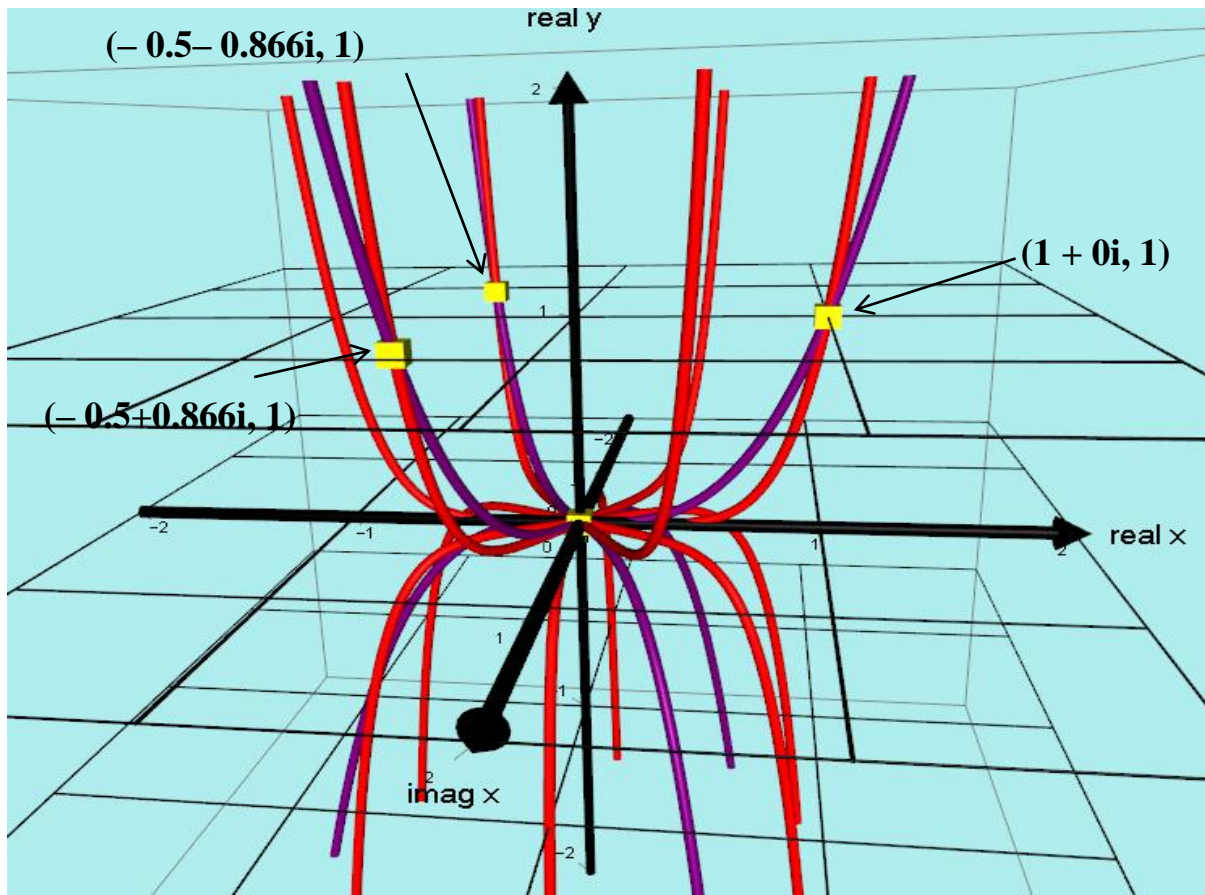
---

Most people just put  $x^6 - x^3 = 0$   
 $x^3(x^3 - 1) = 0$   
 $x = 0$  or  $1$

Some of the better people carried on and put if  $x^3 = 1$  let  $x = r \operatorname{cis}(\theta)$   
 So that  $r^3 \operatorname{cis}(3\theta) = 1 \operatorname{cis}(360n)$  by using De Moivre's Theorem  
 This means  $r = 1$  and  $3\theta = 360n$  so  $\theta = 120n = 0^\circ, 120^\circ, 240^\circ$   
 (actually most people use radians but I work better in degrees)

The solutions are:  $x_1 = 1 \operatorname{cis}(0) = 1$   
 $x_2 = 1 \operatorname{cis}(120) = \frac{-1}{2} + \frac{i\sqrt{3}}{2}$   
 $x_3 = 1 \operatorname{cis}(240) = \frac{-1}{2} - \frac{i\sqrt{3}}{2}$

**So my special contribution was to draw the graphs  $y = x^6$  and  $y = x^3$  with their *phantoms* which shows all the solutions clearly.**



**Incidentally, because of the Fundamental Theorem of Algebra there are 3 more solutions at the origin. The graphs actually cross 3 more times there.**