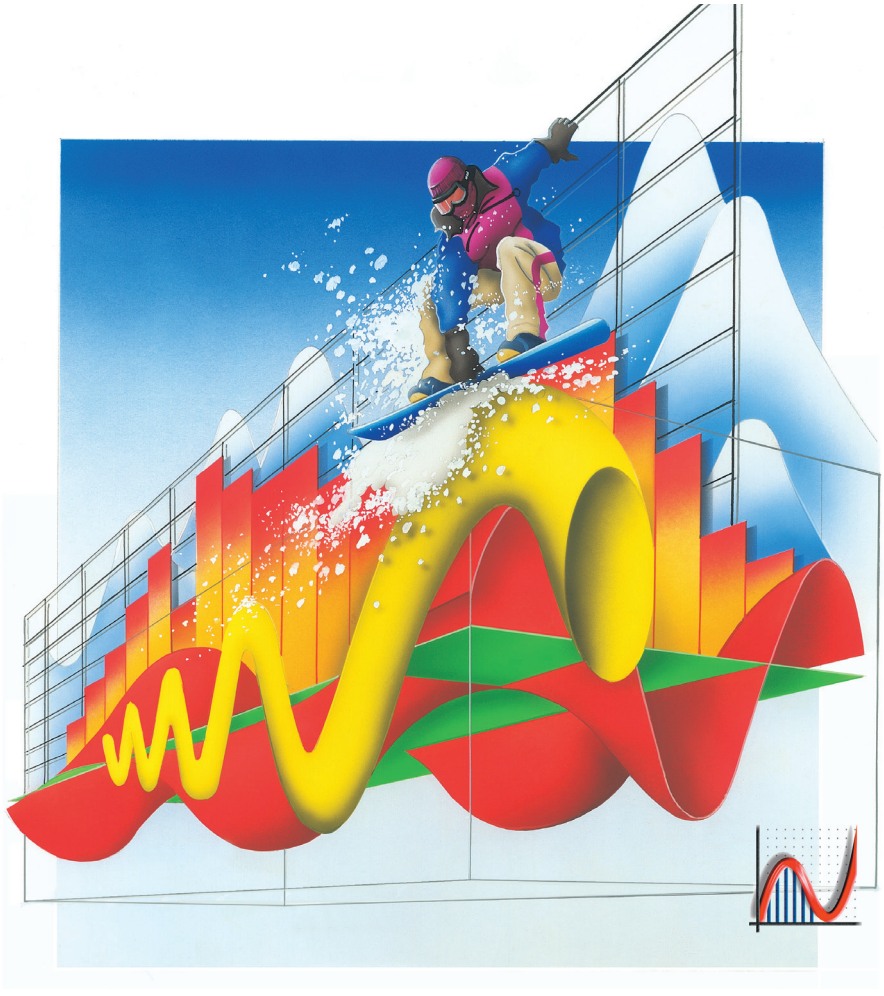
Autograph
version 3



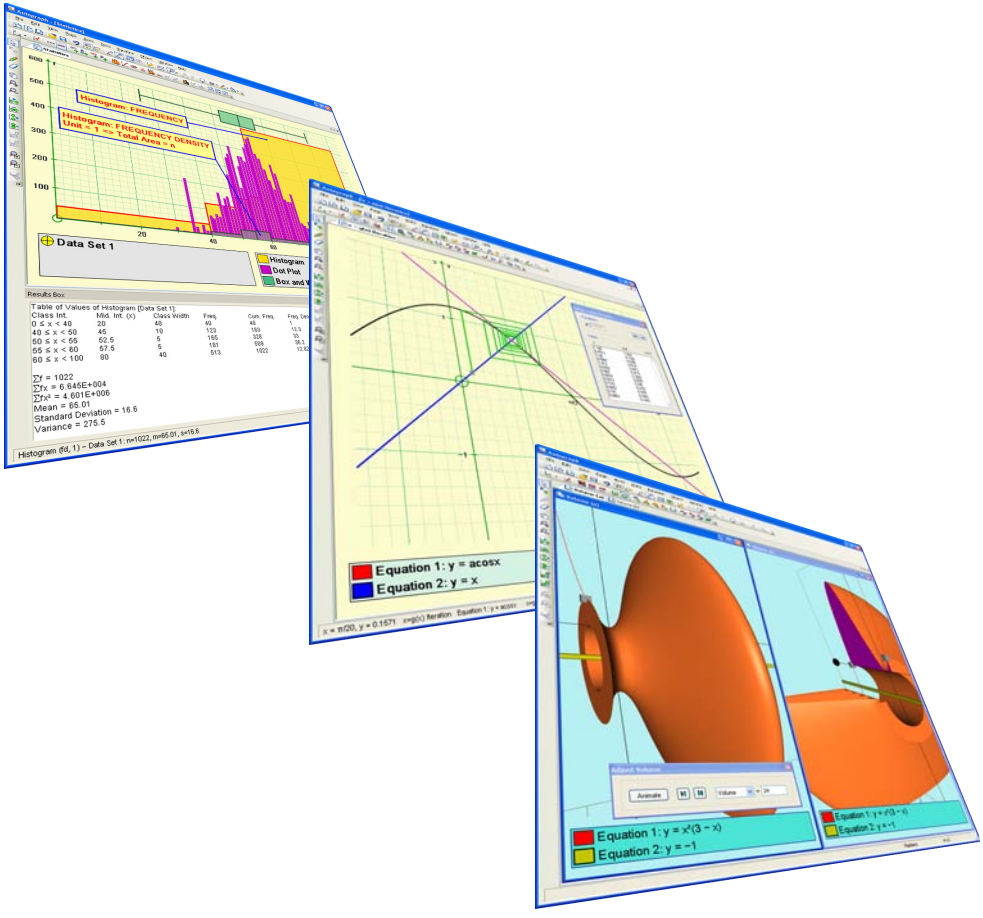
Getting Going with Autograph 3

LINK to v.4 Extension

Douglas Butler and Mark Hatsell

Getting Going with Autograph 3

Autograph version 3



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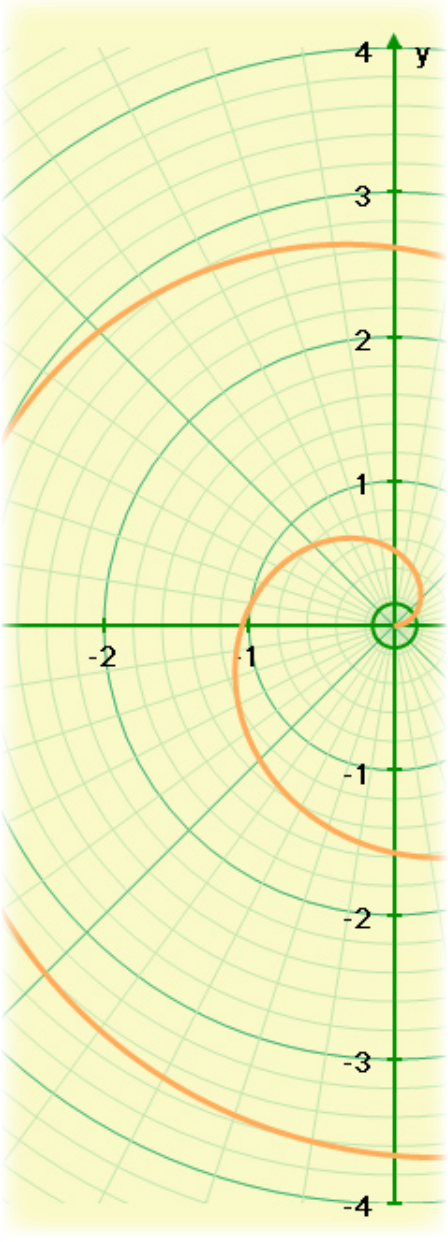
Getting Going with Autograph 3

by
DOUGLAS BUTLER
iCT Training Centre
Oundle School, Peterborough, UK

Autograph is
designed and programmed by
MARK HATSELL

Contents

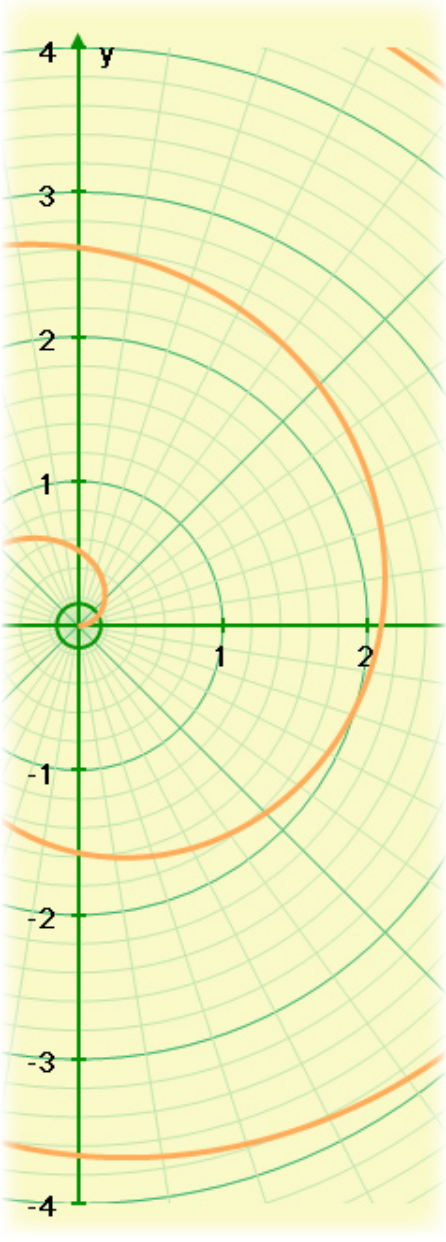


Getting Going with Autograph

1D PAGE: Statistics and Probability

2D PAGE: XY Graphing

Contents



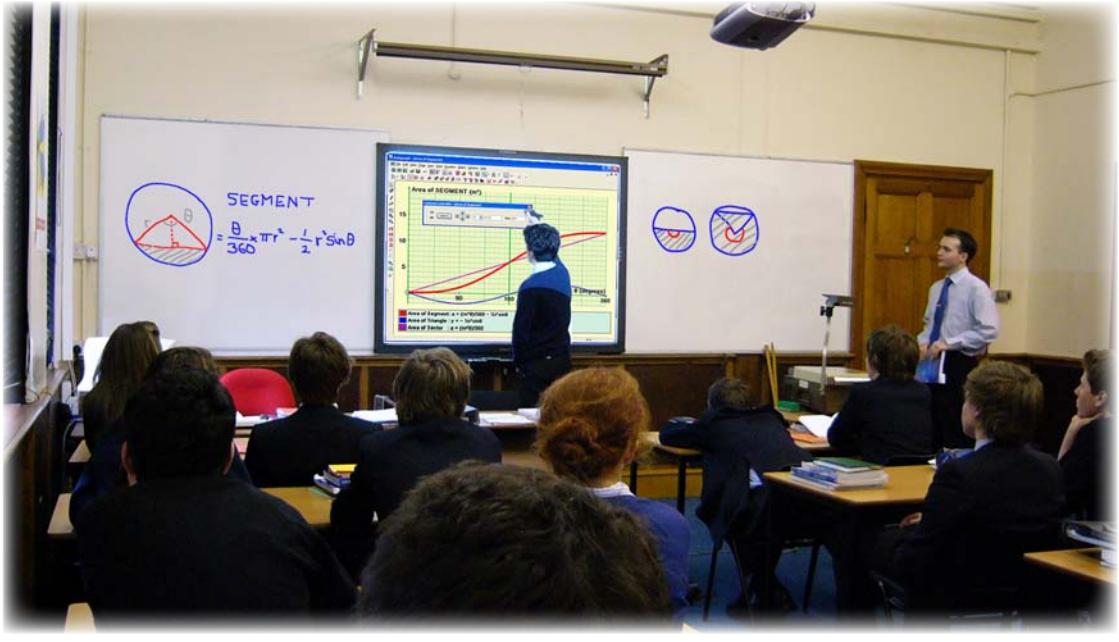
About Autograph

There is now plenty of evidence to show that teaching secondary and college level mathematics with dynamic software is more effective, more efficient, and above all more enjoyable (for both teacher and student).

Autograph 3 leads the way in the use of dependent, selectable mathematical objects to help students get to grips with the basic principles of probability and statistics, and of coordinate geometry in both 2D and 3D. Autograph evolved in the mathematics classrooms of Oundle School (UK), and this 3rd version has come of age to embrace all the possibilities now offered by data projectors, interactive whiteboards and 'tablet' laptops.

There are two levels of operation, 'Standard' and 'Advanced'. The 'Standard' level has a greatly simplified interface and a reduced set of options for the less experienced user.

Autograph uses the standard conventions of Windows applications with regard to multiple windows (called 'pages' in Autograph), the loading and saving of pages as files and the copying and pasting of text, data and graphics.



Launching Autograph

Installation issues are summarized on cards in the box, so hopefully it can be assumed at this point that Autograph is successfully installed!

To launch Autograph, double-click on the desktop icon, or you can launch Autograph from Start => Programs => Autograph 3.20

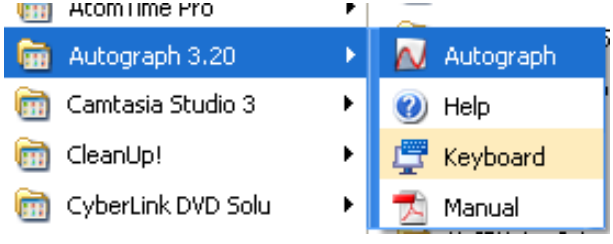
First you will meet the Level Selector:

STANDARD LEVEL - this has been designed for ages 11-16. The interface is simpler (larger icons), and you cannot move out of degrees into radians.

ADVANCED LEVEL - the interface embraces many more options for the discerning user, including calculus, probability distributions, and equations in 3D.

If you click “Don’t show this again” you can reset this in “View” => “Preferences” => “General”

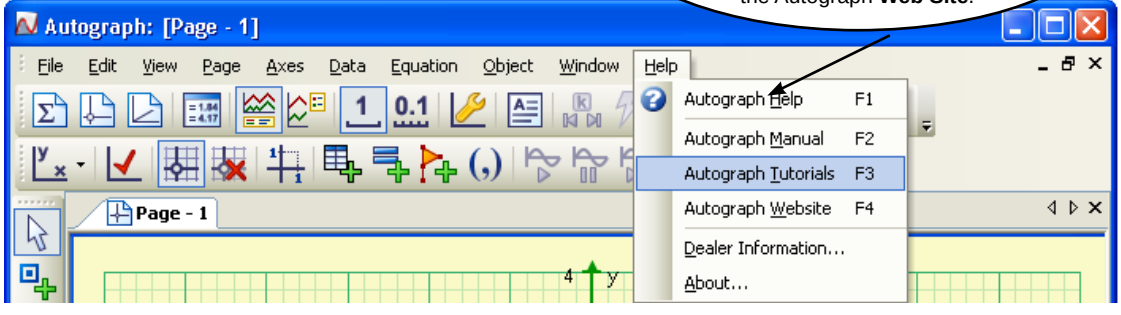
START => PROGRAMS => Autograph 3



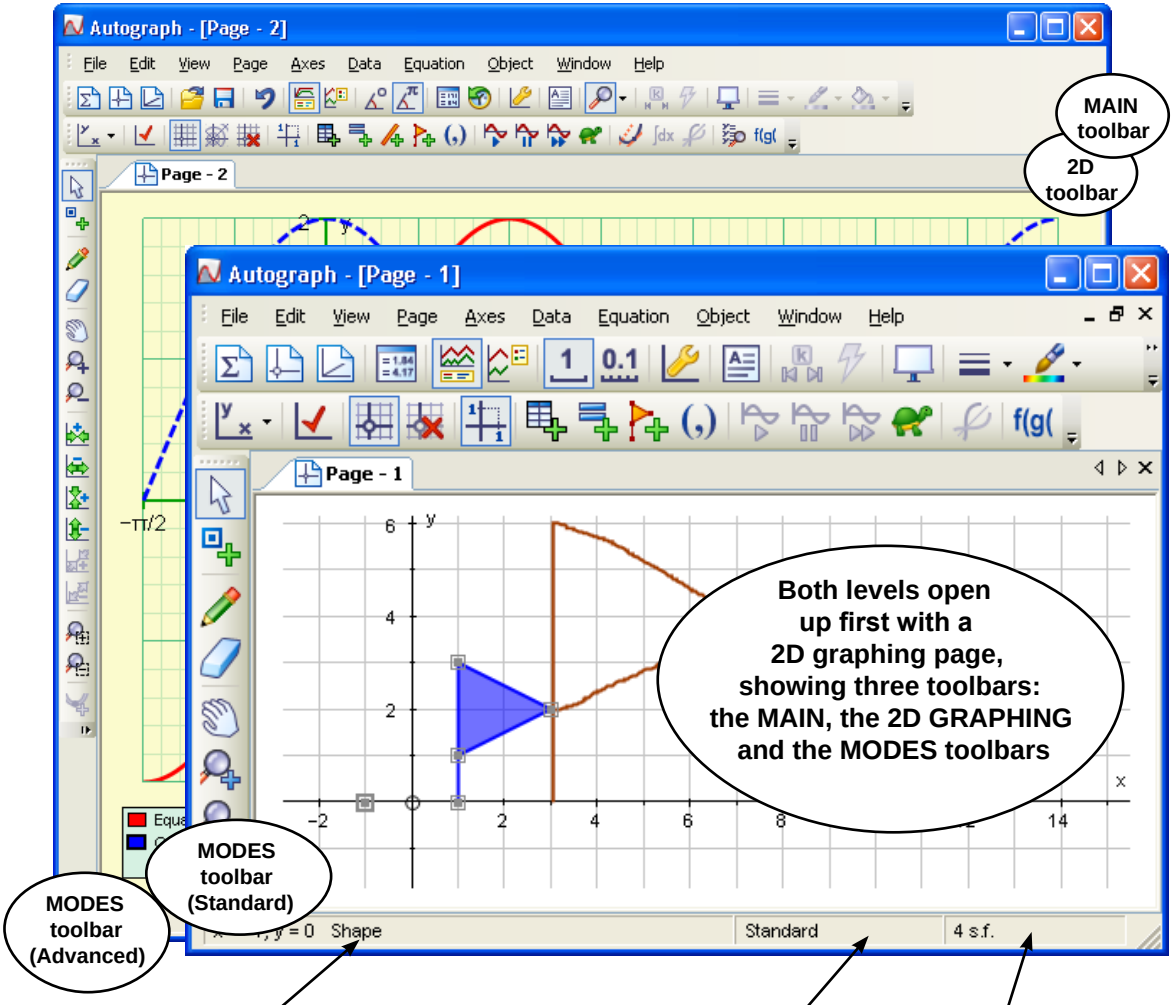
This is the alternative route to launch Autograph, and it is also the location for:

The Autograph Help File (see p. 77)
The Autograph Keyboard (see p. 11)
The Autograph Manual (as a PDF file)

THE HELP MENU



Standard and Advanced Levels



STATUS BAR

This shows **dynamic information** about the currently selected objects, e.g. coordinates, equations.
This information is offered when you open a Text Box. To show the information in a larger free-floating **Status Box** double-click on the bar. To **hide** the bar, go to the View menu and untick “Show Status Bar”.

“Degrees/Radians”

In the Standard level, this just says “Standard”. In Advanced level, this indicates “Radians” or Degrees”, as set locally in “Page” => “Edit Settings”.

“4 s.f”

This indicates the accuracy currently being used for all results, coordinates, etc. Go to “Page” => “Edit Settings” to amend locally.

The Main and the Modes Toolbars

THE MAIN TOOLBAR


Add new **1D Statistics Page**


Add new **2D Graphing Page**


Add new **3D Graphing Page**


Open new **Autograph file** (.agg) [or FILE menu]


Save current page as .agg file [or FILE menu]


Undo [Ctrl-Z]. Also: **Redo** [Ctrl-Y]


Equation/Object **key** at the bottom


Equation/Object **key** at the side


Integer snap [Standard level only]


0.1 snap [default in the Advanced level]


Angles in degrees, radians [Advanced only]


RESULTS BOX (a text file of recent results)


History of equations (2D and 3D)


EDIT AXES (see p. 12)


TEXT BOX (offers text from the Status Bar)


MODES TOOLBAR on/off


CONSTANT CONTROLLER (see p. 14)


ANIMATION CONTROLLER (see p. 15)


WHITEBOARD MODE (see p. 11)


Line thickness (of selected object)


Line colour (of selected object)


Fill colour (of selected object)

THE MODES TOOLBAR


SELECT MODE. Selects, when over an object. Otherwise:
2D: *Drag a rectangle* round points to select them.
3D: *Acts the same as the Drag mode*.

ESC will always return you to this mode.
ESC again deselects current selection.
Pressing '.' will temporarily put you in **'Point' mode** while you hold it down, then return you to 'Select' mode - useful for placing single points.


POINT MODE: place free points, or on objects

+ Ctrl: Finds intersections at f(x)=0 or f(x)=g(x). The mouse changes to a small circle when over an intersection.


SCRIBBLE MODE: generates writing objects which can be selected (to change colour, thicken, delete, etc). [Not available in 3D.]


RUB-OUT: like a board rubber - everything in its path is deleted. Use **Ctrl-Z** (undo) to bring them back!


DRAG MODE.
1,2D: Drags the whole axes; everything is redrawn.
3D: Rotates the scene
+ **Ctrl:** UP/DN: moves the 'camera' in and out
+ **Shift:** moves the 'camera' about.


ZOOM-in (centred where you click)


ZOOM-out (centred where you click)


Rectangle ZOOM in [not available in 3D].

The full set of zoom modes is available in the Standard level from the 'Axes' menu => 'Zoom Modes'

More on the Advanced Level Modes Toolbar:


ZOOM in/out in one direction only: 'x', 'y' or 'z' (3D only).


Rectangle Zoom OUT: putting the whole scene into the drawn rectangle [not available in 3D].


This tool turns on automatically when solving a differential equation or integral function.

The Autograph Menus

FILE MENU

New 1D Statistics Page
See page 21

New 2D Graphing Page
See page 39

New 3D Graphing Page
See page 65

New Extras Page:

- **Area of Circle**
- **Trigonometry**
- **Dice Simulation**
- **Monte Carlo**
- **Confidence Intervals**
- **Poisson Grid**

See page 82

Open (saved **.agg** file)
○ **Close** (current page)

Save ... ... **[Ctrl-S]**
○ **Save As**

Print Preview
Print ... **[Ctrl-P]**
○ **Print Setup**
Refers to the current page

Exit

NOTE: AUTOGRAPH FILES (.agg):

To save any 1D, 2D or 3D Autograph page (file extension ".agg") use "File" => "Save" [Ctrl-S] or "Save As", or the "Save" icon on the main toolbar.

To load a ".agg" file either double-click on it (to start up Autograph if it is not already running), or drag it onto an open Autograph window, or use "File" => "Open".

Autograph v.2 saved its files in two separate formats:

2D Graph files: ".agg",
1D Statistics files: ".ags"

Both types can be loaded into V.3, but files saved from V.3 are all ".agg" files. V.3 files will not load into V.2.

EDIT MENU

Undo ... ... **[Ctrl-Z]**
Redo ... ... **[Ctrl-Y]**

There is unlimited Undo/Redo for each page that is open.

○ **Select All Points**
○ **Select All Scribbles**
○ **Select All Objects** **[Ctrl-A]**

VIEW MENU

Constant Controller
See page 14

Animation Controller
See page 15

○ **Status Bar** (on by default)
○ **Status Box**
See page 16

Results Box
See page 16

○ **Instructions Window**
A page of instructions as edited and saved in "Page" => "Edit Instructions"

Onscreen Keyboard
See page 11

○ **Toolbars**
○ **Load/Save Toolbars**

Operation of the toolbars is automatic, and they are generally best left alone! However you can use these options to select, customize and reset the toolbars. You can load and save particular toolbar arrangements.

○ **Preferences**
See page 10

PAGE MENU

○ **Edit Settings**
All these settings are local to the current page. You can name the page, associate a file as the Instructions page, set the angle units [Advanced only], and set the accuracy (default 4 sig fig).

○ **Edit Instructions**
You can create and save a page of instructions for the current page.

○ **Reset Page**
A list of all the reset options on the Statistics, 2D and 3D page toolbars.

○ **Copy Page (Bitmap) [Ctrl-C]**
Places the graphing area and key onto the clipboard, This can be a large file, so it is better to reduce the Autograph window size first.

○ **Copy Graph (Metafile)**
This copies the graphing area as a high quality 'vector' image to paste into Word. [Not available in 3D.]

○ **Copy Equations** (text)
○ **Copy Status Bar** (text)
○ **Save Page** (bitmap)
○ **Save Graph** (metafile)

AXES MENU

Edit Axes
See page 12

- **Snap Settings**

Use this to set up how a point moves when its position is altered using the arrow keys. The default settings should suffice for most situations.

- **Zoom Modes**

The full set of zoom modes are listed here, the same as on the Advanced Modes toolbar. Not all are in the Standard Level Modes toolbar, but all are available in both levels

Move Origin
Moves the origin to where you click.

Move Centre
Moves the centre of the page to where you click.

Select Mode
ESC will return you to this state.

Point Mode

- **Show/Hide Equation Key.**

Use this if the key is unnecessary, or to retrieve it. This option has been added to the 'key' right-click menu.

DATA MENU

Enter XY Data
See page 52

Enter Coordinates
See page 42

Enter Shape
See page 50

Enter Grouped Data

Enter Raw Data
See page 25

Enter Box and Whisker Diag.
See page 28

Enter Probability Distribution
See page 33

EQUATION MENU

This is a duplicate of the right-click menu when the mouse is over the Equation Key on a 2D or 3D Graph page.

Enter Equation ... **[Enter]**

Edit Equation
See page 54

Delete Current Equation

Delete All but current

Delete All Equations

Function Definitions
See page 54

Manage Equation List

Here you can add, delete and edit equations in the list. You can also paste from the History list, and choose to hide/unhide equations.

View History

This lists all 2D/3D equations that have been plotted in the current session. Any selection from the list can be plotted.

- **Create (current equation):**
  - **Gradient Function**
  - **Integral Function**
  - **Reflection in y=x**

See page 57

OBJECT MENU
[= Right-click Menu]

This is a duplicate of the main graphing area Right-Click menu, which lists the appropriate options for the objects selected on the current page. This is useful when working on an Interactive Whiteboard, or on a Mac computer (using 'Virtual PC').

WINDOW MENU

- Tabbed Workspace
  [ticked ON by default]

Cascade F5

Tile Vertically F6

Tile Horizontally F7

- **Arrange Icons** ... ...
- **Close All** ... ...

- **List of open windows**
  browse: F8 <=> F9

HELP MENU

Help Contents F1
See page 77

- **Autograph Manual (PDF)** F2
- **Autograph Resources** F3
  See page 79
- **Autograph Web Site** ... F4
- **About Autograph**

Checking your Preferences

Before you get going, look at the **Preferences** (in the '**View**' menu) and check that they are set up the way you want them. There are four "TABs":

♦ PLOTTING

Options
Older computers will struggle with Autograph's sophisticated plotting style which makes lines appear smooth. You can try switching to 'Normal' or, for really old machines, to 'Fast'.

Line Thickness
Use "Thick Lines" for demonstration work. 2¼ pt is a useful thickness, applying to all graphs and lines.

♦ GENERAL

Inequalities
Set the shading you prefer for illustrating inequalities.

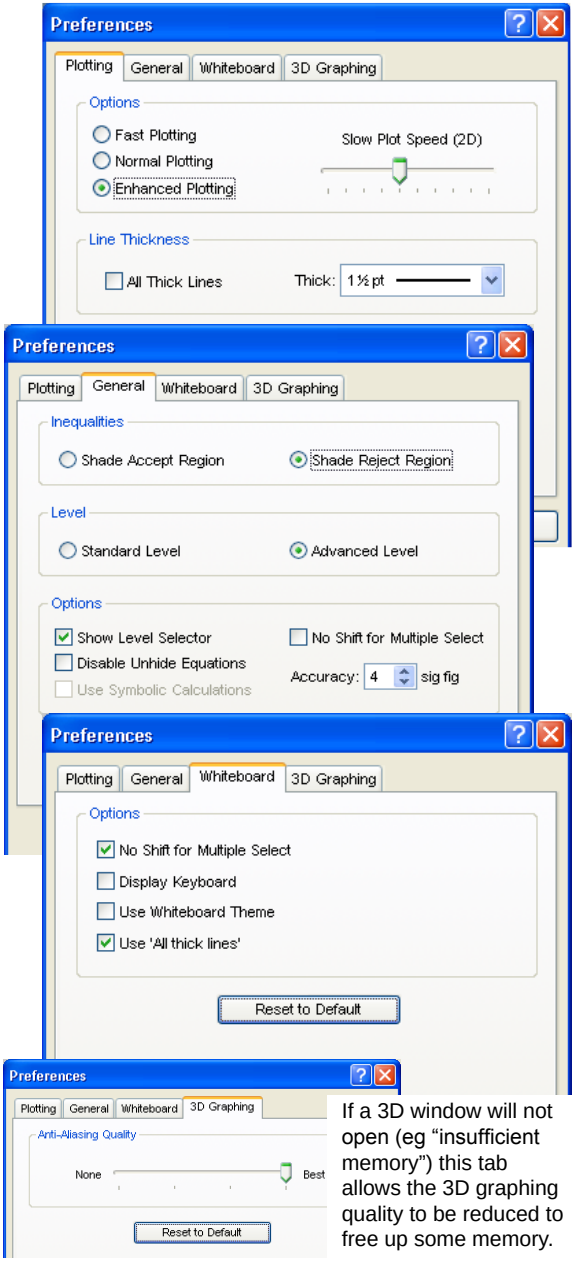
Level
This is the path to switch between "Standard" and "Advanced" without restarting Autograph.

Options
Show Level Selector: This is how to ensure that the level selector shows on start-up.
Disable Unhide Equations: This disables the 'Unhide' option in Equation => Manage List.
Accuracy: this starts at 4 sig. fig., but can be reset here as the start accuracy for all pages opened by this user.

TO SHIFT OR NOT TO SHIFT?
It is standard Windows practice to use the SHIFT key to enable the selection of more than one object on the screen. With the advent of Interactive Whiteboards, it is more sensible to accumulate selections without pressing Shift. The default setting is to use "Shift". Tick the box here to apply "No Shift" to all pages.

The downside of "no shift" is that any previous selection may still be active. It is recommended therefore before any new selection, you click on the graph area to de-select any residual selection [pressing "Escape" also does this].

♦ WHITEBOARD
Here you set the Whiteboard Mode preferences. Note: the default is "No Shift for Multiple Select" when in Whiteboard Mode.



If a 3D window will not open (eg "insufficient memory") this tab allows the 3D graphing quality to be reduced to free up some memory.

Using an Interactive Whiteboard

There are two main types of projector-driven Interactive Whiteboards. Both provide teachers with a large screen which they can control directly while standing at the board:

1. Pressure Sensitive Boards (e.g. 'Smart' Boards from Canada)
Nothing happens until the board is pressed, so there is no helpful 'mouse cursor', but it has the advantage that anything can act as your 'pen', e.g. your finger, a drumstick, etc. Right-click is achieved by pressing a button on the tray, then tapping on the board. The lack of mouse cursor suggests that there will be times when a regular keyboard and mouse should be used for constructions, then go to the board for any animations.

2. Boards, Graphics Tablets, or 'Tablet' PC laptops with an electronic 'pen'
These use a pen which broadcasts its position to the screen, and you get the important 'mouse cursor'. Right click is achieved by pressing a button on the side of the pen. Tablets allow the teacher to control the image from anywhere in the classroom, including handing over to the pupils.

Autograph's IWB friendly features:

WHITEBOARD MODE - click on this to switch on:
- No Shift for Multiple Select - Display Keyboard - Whiteboard Theme (larger letters, thicker axes)
- All thick Lines, all as set in "View" => "Preferences" => "Whiteboard".

SCRIBBLE MODE - this will add writing directly to the graphing page. This is much better than an IWB writing tool: it does not disappear on switching back, and it 'belongs' to the page, so will rescale if the axes are altered. Select a scribble by clicking on it, or use "Edit" => "Select all Scribbles", to drag, change colour or thickness, or delete the writing.

RUBOUT TOOL - use this as you would a board rubber. Any object it touches will be deleted. Anything you delete in error can be revived using "Edit" => "Undo", or Ctrl-Z.

ON-SCREEN KEYBOARD - this can be turned on and off in "View" => "Keyboard". With its minimum configuration (right) you can manipulate points, objects and selections. "**Shift**" and "**Ctrl**" stay set until cancelled. Use "**Esc**" to return to the "Select" Mode, and "**Esc**" again to clear any object selection. "**PgUp**" and "**PgDn**" are used in the Constant Controller to switch between constants. "**Undo**" is included, so is "**PtSc**" which copies a bitmap of the screen.

Data designed for data entry and axis ranges: includes π , TAB and – (minus) and comma (for lists)

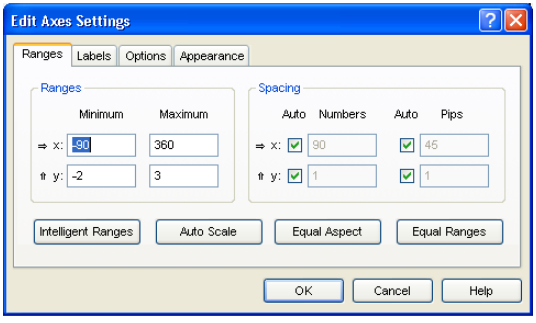
Text designed to include common mathematical symbols and to **eliminate the need for "Shift"**. Keys such as "<", ">", "+", "(", ")" are all Shift keys, so these have their own buttons, along with "sin", "cos", "tan", some common indices (including $^{-1}$, used for inverse sin, etc.) and symbols.

Extra The Keyboard uses "*Arial for Autograph Uni*" font *(see p.80)*. The 'extras' includes Greek letters that can also be used in an Equation as constants.

e.g.:
Text: (x̄, ȳ), ∫sin²θ dθ
Equation: y = sin(x + α)

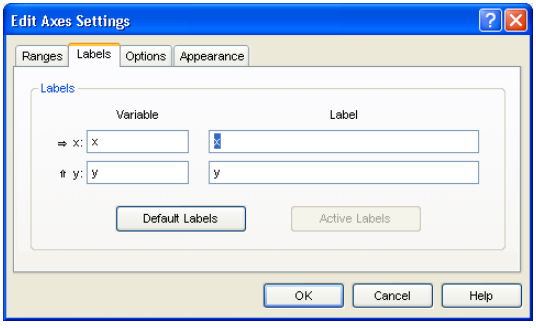
Sorting Out the Axes

EDIT AXES - There are many ways to customize the Autograph pages, and what is described here is much the same for 1D, 2D or 3D pages.



RANGES TAB: this allows the min and max values and pip spacing to be set automatically or manually. Pressing any of the four buttons shows the resulting scales on offer, so you can check before you "OK".

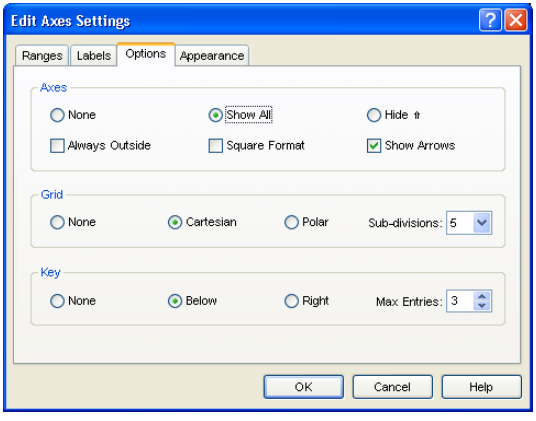
Use the DEFAULT SCALES button to set appropriate scales for what is plotted (same as "Intelligent Ranges"). In particular for 2D trig functions, the scales respond to the degrees/radians setting.



LABELS TAB: Labels can be any text.
The variables can be any single letter e.g. 'v', 'Velocity (ms^{-1})' and 't', 'Time (s)', when equations such as v = 2t(1 − t) can be entered.

This 2D toolbar button sets 'x' to 't' and 'y' to 'x'.

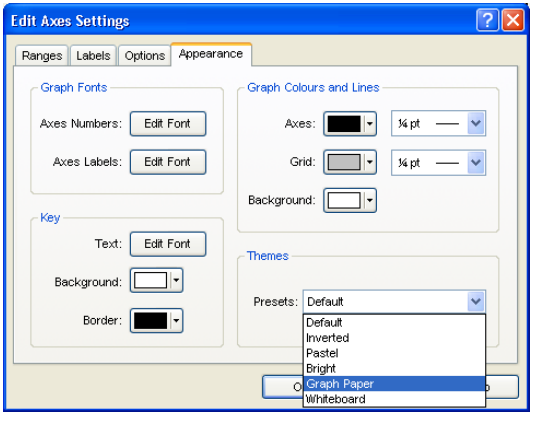
This 2D toolbar hides the 'y' axes to turn the 'x' axis into a number line.



OPTIONS TAB:

Axes - use "Hide ⇑" in 1D to take off the 'f' axis (eg Box and Whisker), in 2D for number-line work.
Grid - use "sub-division" for the graph-paper effect.
Key - use this to control the equation key.

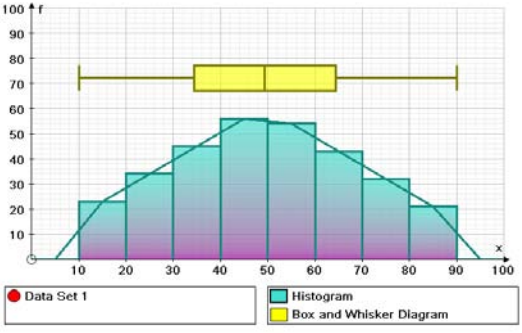
3D: is set "Always Outside" by default.



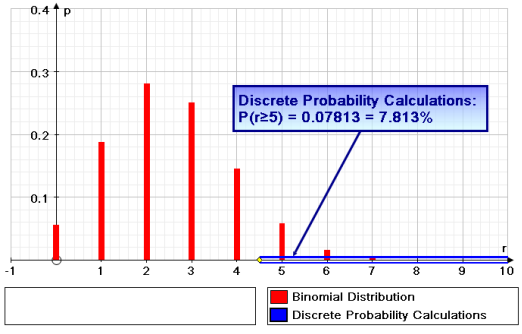
APPEARANCE TAB:

Here you have complete control over the font in use, the colour of the axes and grid, and the background colour. Use the presets to instantly set up a range of settings, e.g. "Graph Paper".

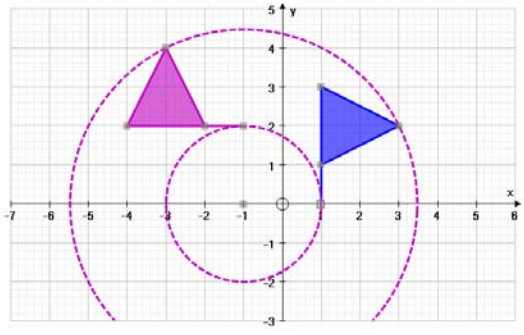
3D: this tab includes control over transparency.



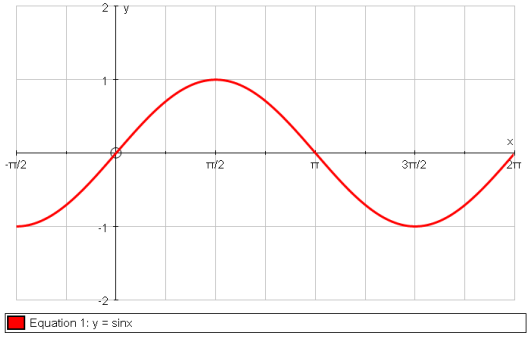
1D: The default axes for displaying data. The key has two sections, one for data sets, one for objects.



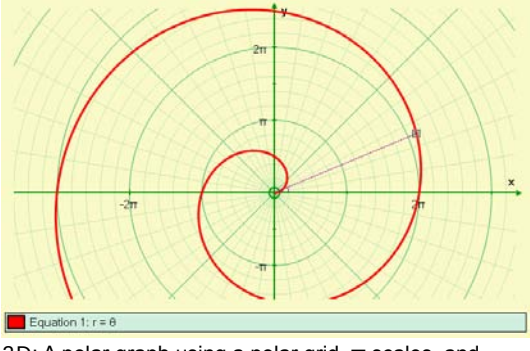
1D: Axes for displaying probability distributions. The labels are changed automatically, but can be edited.



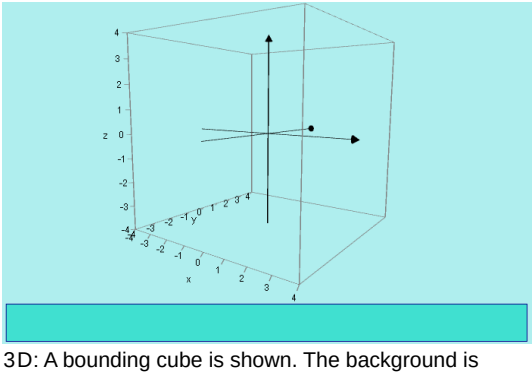
2D: Axes for displaying 2D graphs. The key has been turned off ("View" menu).



2D: Using the "Default Axes" button for a trigonometric graph, puts in π scales when angles are in Radians.



2D: A polar graph using a polar grid, π scales, and equal aspect.



3D: A bounding cube is shown. The background is black by default, but can be made lighter.

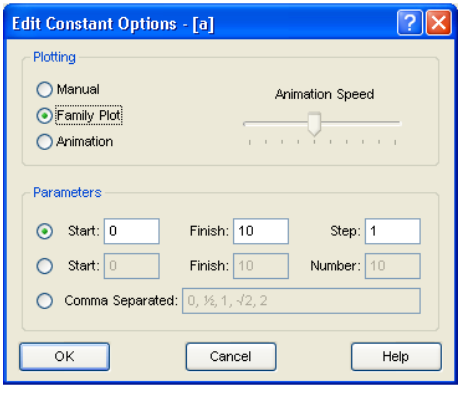
The Constant Controller

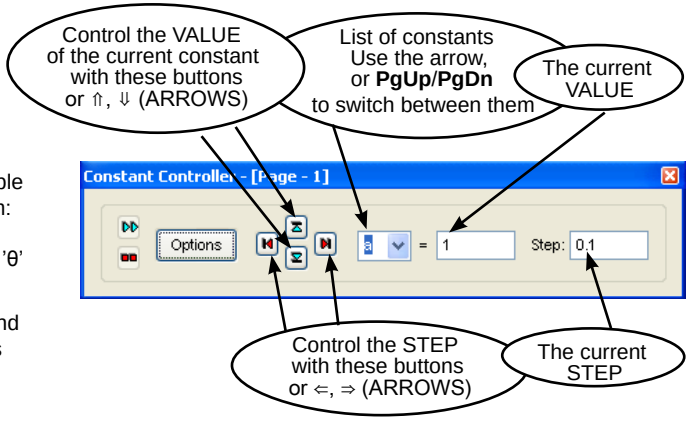
The CONSTANT CONTROLLER is used in 2D and 3D pages to control constants that have been included in: **Equations,** e.g.: y = mx + c, **coordinates of points**, e.g. (a, b), **vectors**, etc.

Permissible constants include all the available Greek letters, and all normal letters apart from:
'e' [reserved for the exponential constant]
'i' and 'j' [reserved for complex numbers] 't' or 'θ' may be used when they are not parameters.

The constant controller 'belongs' to a page, and controls all constants on that page. Constants can each occur more than once and in more than one equation or object.

Values of a selected constant are best controlled dynamically by the UP/DOWN arrow keys, and the **step** by the LEFT/RIGHT keys.



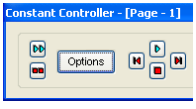


OPTIONS

Manual: direct control of one constant at a time.

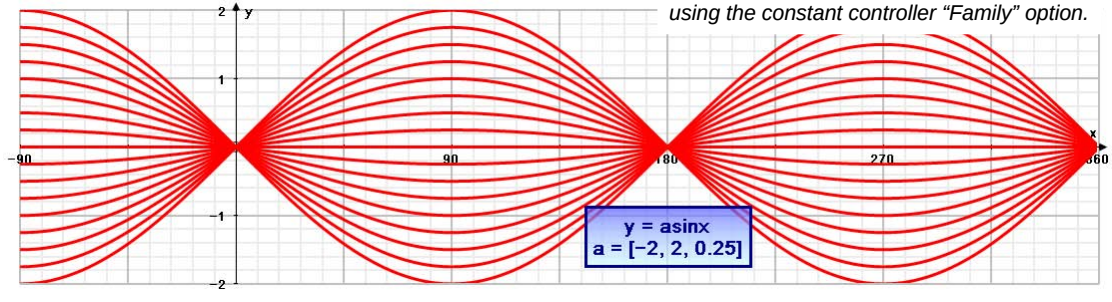
Family Plot: *this affects constants in 2D graphs only.*
You can set any number of graphs to plot at the same time with a range of values, including a comma-separated set, of the chosen constant.

Animation: an animation of the constant can be set up, and this can apply to constants in 2D or 3D equations, and in coordinates and vectors in 2D and 3D.



With an animation set up the control buttons change to '**Go/Pause**' and '**Stop**'. The pair at the left end are used when a number of animations are set up, so you can start them all off at the same time.

A family of curves drawn from y = asinx, using the constant controller "Family" option.

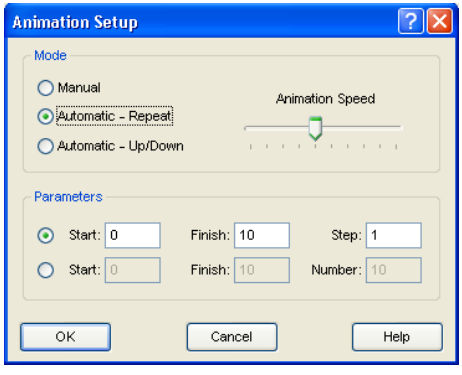


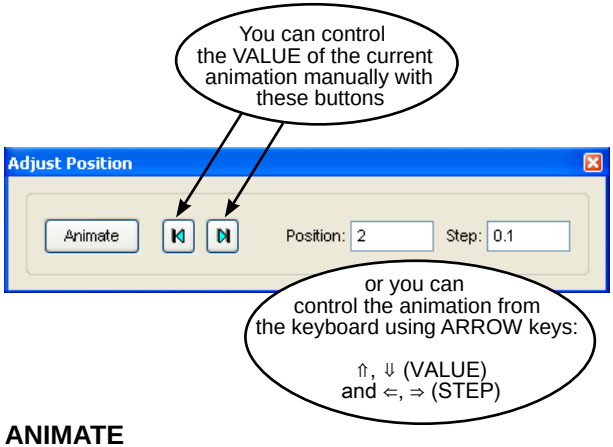
The Animation Controller

The ANIMATION CONTROLLER is used to animate the parameter(s) of an operation, for example:

1D: 'n' and 'p' for a binomial distribution
the class width for a grouped data set
2D: transformation, enlargement scale factor
transformation, rotation angle
x-coordinate of a point [y=f(x) graph]
t-coordinate of a point [parametric graph]
3D: transformation, enlargement factor
t-coordinate of a point [parametric graph].

To set up an animation, select an object that has the factor or parameter (e.g. an enlarged shape). Click on the button, which will light up for all objects that can be animated.



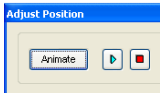


ANIMATE

Manual: direct control of one value at a time.

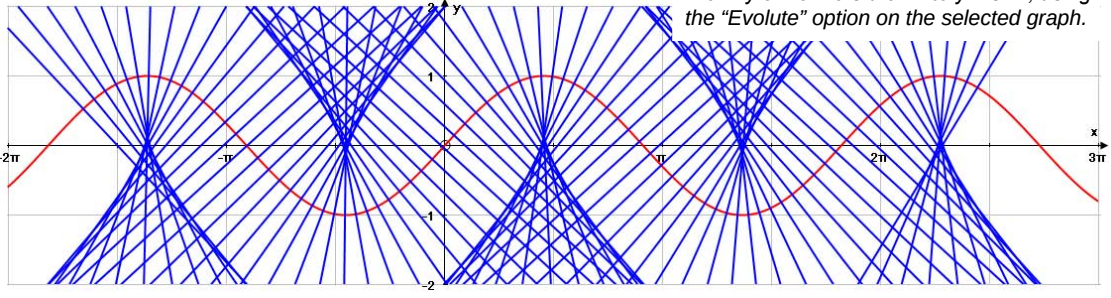
Automatic - repeat: this will animate from the start value to the finish value, then jump back to the start.

Automatic - Up/Down: this will smoothly animate up and down the value set.



With an animation set up the control buttons change to '**Go/Pause**' and '**Stop**'.

NOTE: another way to achieve an animation is to put a constant in as the 'factor' (e.g. enlargement factor), and use the constant controller animation feature. *(The 'family' option is only available for constants that are in equations.)*



A family of normals drawn to y = sinx, using the "Evolute" option on the selected graph.

The Results Box and Status Box

THE RESULTS BOX

The Results box resides in a TAB on the right side by default. If it has been closed you can launch it by pressing the toolbar button, or using "View" => Results Box".

Sometimes the results box is opened by some other action (eg "Transfer to Results Box" or "Display Information"), or you can click on the TAB to open it - it will slide open and not disturb the graphing area, revealing its contents. It will remain open until you click somewhere else. Its **left boundary** can be dragged to the left or right to vary its width as required.

If you click the **pin icon**, you turn it into a dockable window box
- Drag its blue bar to make it float, or 'dock' on another edge of the graph area
- Double-click on the blue bar to toggle its position between 'docked' and 'floating'. However you leave it will be remembered next time to start up.

The results box is a **general repository** for answers to calculations (eg intersections) and statistics results. It is all created in the resident '*Arial for Autograph Uni*' font.

Any selected text in the box can be copied out to another application (eg Word). When pasted, all formatting and font information is preserved. You can use the right-click option "Clear All" to empty its contents, though this is not usually necessary.

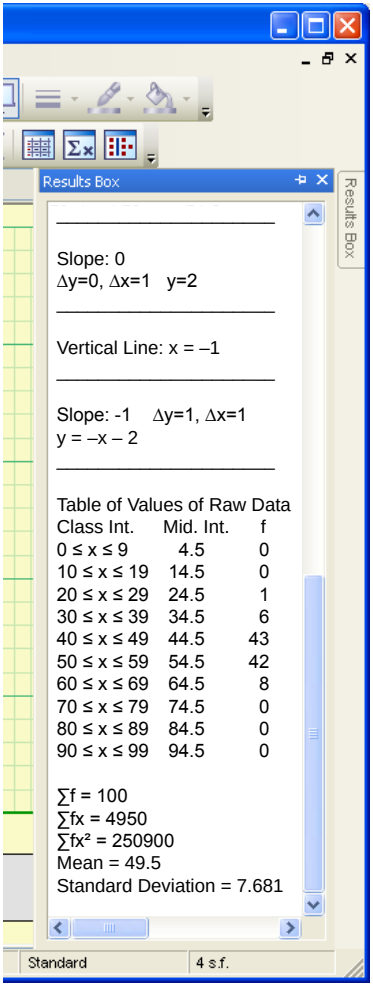
1D Results that are put in automatically include:
Table of Statistics or Probabilities

1D Results that are put in manually include:
Statistics Box (click "Transfer to Results Box")

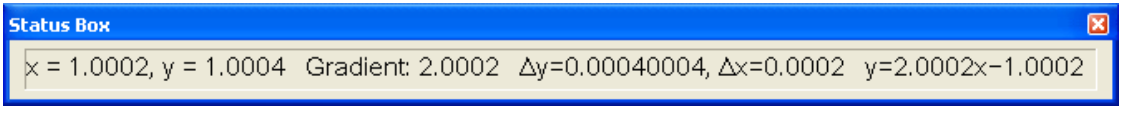
2D Results that are put in automatically include:
Equations of lines (tangents, normals, segments, etc)

2D Results that are put in manually include:
Selected graph: option "Display Information"

3D Results that are put in automatically include:
Intersections: two lines, two planes, three planes.



THE STATUS BOX

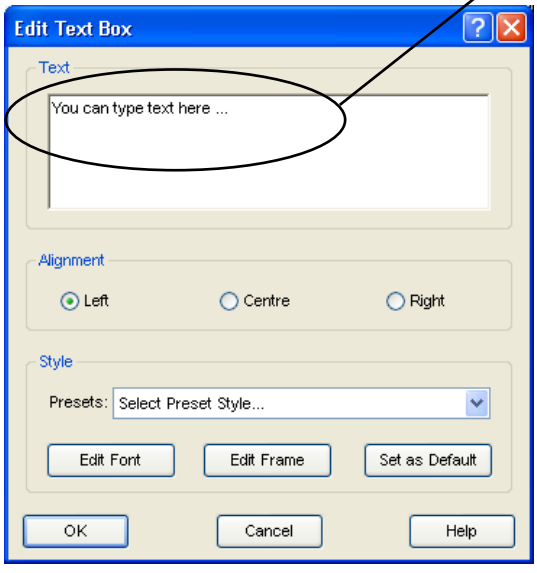


To view the contents of the Status Bar in a moveable and resizable window and with a larger typeface, either double-click on the Status Bar, or use "View" => "Status Box". To close the window, double-click on it or click the cross. The example here shows the dynamic information from a gradient triangle, to 8 d.p. (set in "Page" => "Edit Settings").

Adding a Text Box

TEXT BOX (Static Text)

Use the right-click option or click on the icon in the main toolbar. Due to the restrictions of Direct-X it is not possible to put a text box on a 3D page.

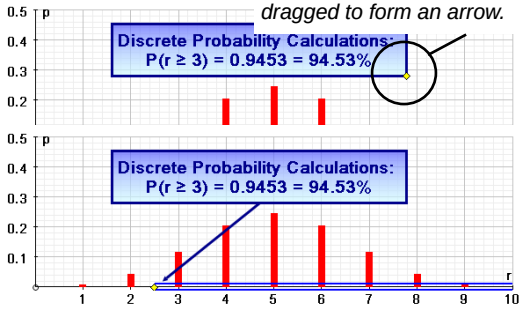


The text from the **Status Bar** placed in the text box when it is opened. This can, of course, be edited as required.

Layout, Font and Frame can all be edited to the usual Windows conventions (including gradient fill), and a number of preset layouts are provided (e.g. “Deep Blue”, used below and the opening ‘default’ style).

• **Set as Default:** This will make your current text box style the default style, and therefore the style of the next box to be created.

The yellow diamond can be dragged to form an arrow.



TEXT BOX (Dynamic Text)

With a single object selected, the text box allows you create a label with text that ‘belongs’ to the object (as in the Status Bar, though this often contains the details of more than one object). The text box can contain two types of text:

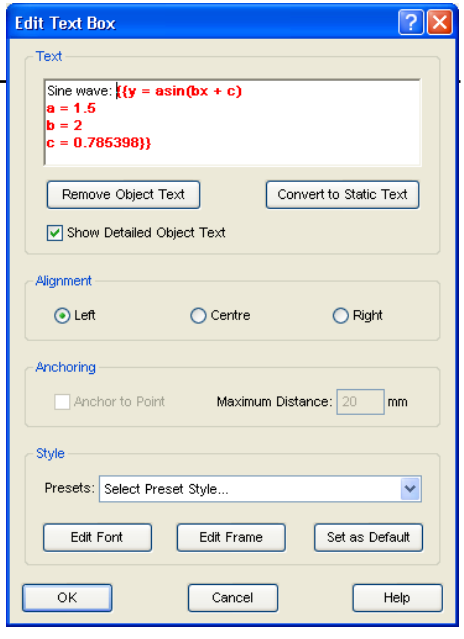
- the Static part, which appears in black and is fully editable, and
- the Dynamic part, which appears in red between {{...}}. This text will change dynamically if the object is moved or edited in any way.

• *Remove Object Text / Insert Object text*
This removes or inserts the dyanmic part of the status bar text

• *Convert to Static Text*
This converts the red text to static, fully editable text.

• *Show Detailed Object Text*
This shows dynamic values of any associated constants in use.

• *Anchoring [only relevant for points]*
Anchor to the object, or not, and set the distance from the object .



Edit Draw Options

Most objects can have their appearance defined by this dialogue box, from the Right-click menu for one or more selected objects.

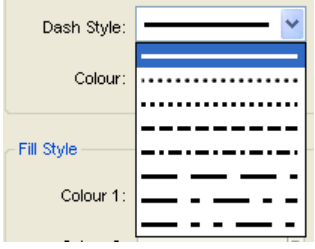
If more than one object is selected all objects will be forced to take the same style.

LINE STYLE:
This can be applied to any object which is, or contains, a line.

Line Thickness:

This can also be set by  on the main toolbar

Dash Style:



This can be applied to any graph or line or vector

Line Colour:

This can also be set by  on the main toolbar.

FILL STYLE: (eg Shapes and Histograms)

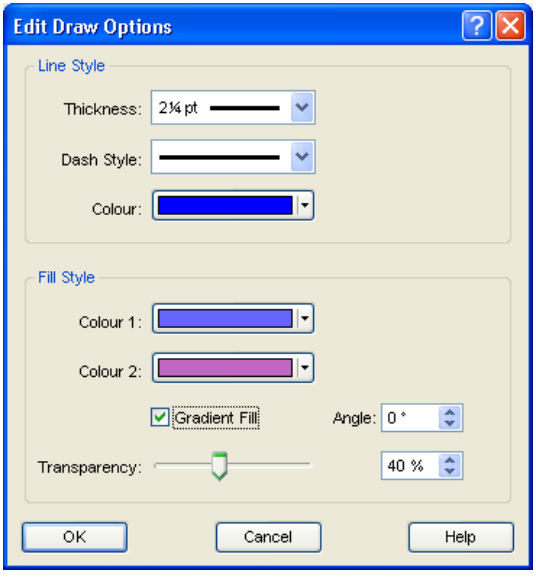
Fill Colour 1:

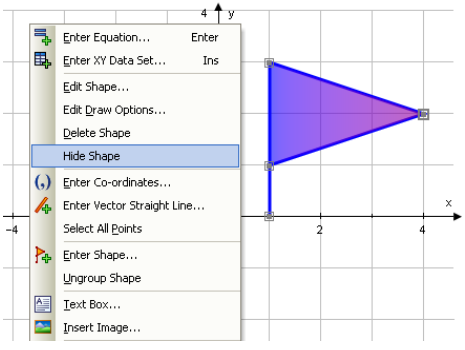
This can also be set by  on the main toolbar

Fill Colour 2:
This provides the second colour if "**Gradient Fill**" is ticked. *[not available on a 3D page]*

Angle:
This sets the angle of the gradient fill.

Transparency:
Set the % using the slider or by entering a value.
0% = solid; 100% = fully tranparent.





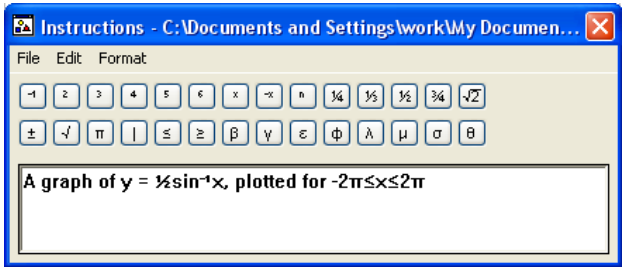
HIDE / UNHIDE OBJECTS

Hide: One or more selected objects can be hidden, using the right-click option "Hide"

Unhide: If you can find and select a hidden object, you can use the right-click option "Unhide object"

Unhide All: This right-click option will unhide all hidden objects.

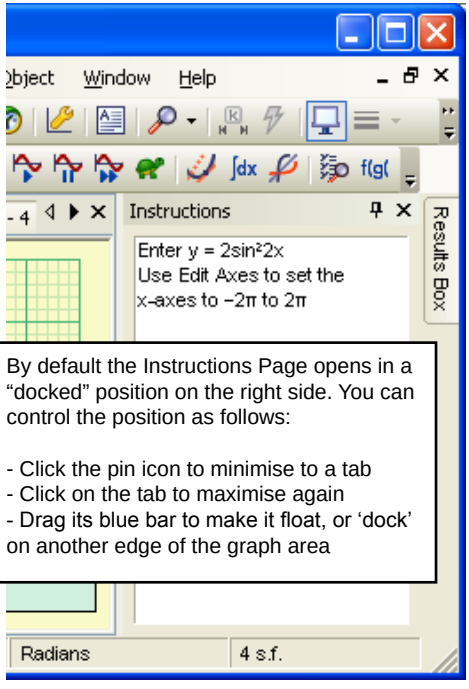
Creating an Instructions Page



Use Page menu => Edit Instructions. All the Autograph characters are available. There is font control [though the Arial for Autograph font is required if the symbols are used] and formatting control (left/centred/right).

Use File => Save, and wherever this Instructions file is saved, it will be associated with the current Autograph File (if it is also saved). When the current Autograph file is reloaded, users can reload and view the associated Instructions file, using "View => Instructions Window".

You can also check where the Instructions file is located in the Page Menu => Edit Settings.



By default the Instructions Page opens in a "docked" position on the right side. You can control the position as follows:

- Click the pin icon to minimise to a tab
- Click on the tab to maximise again
- Drag its blue bar to make it float, or 'dock' on another edge of the graph area

USING AUTOGRAPH: SOME BASIC PRINCIPLES

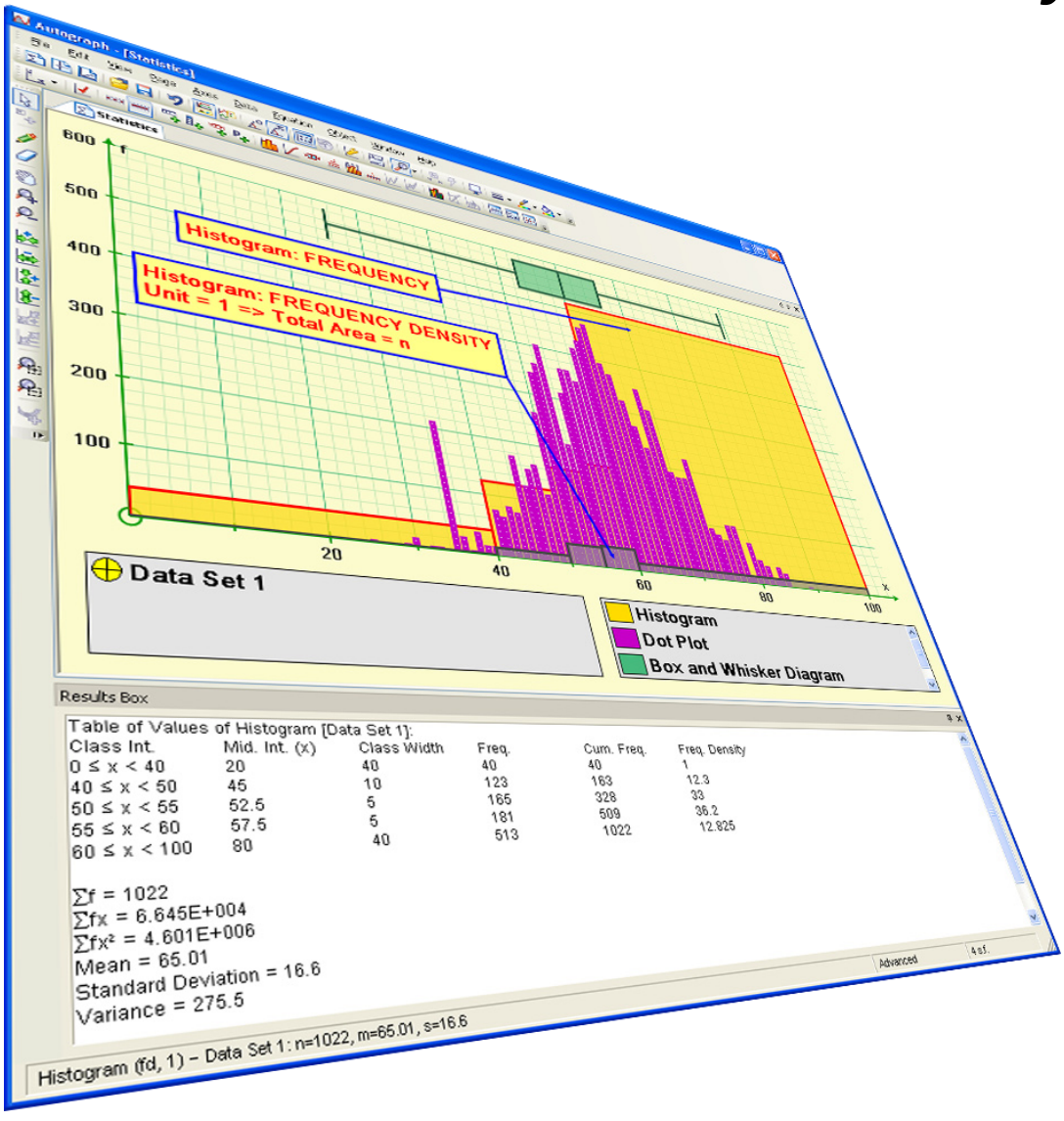
Use the RIGHT-CLICK menu:
- to create top level objects (e.g. a graph).
- to find the appropriate operation to carry out on the current object selection.

Selections: with or without SHIFT:
- check this out in "View" => "Preferences".
- The default is to use SHIFT for multiple selections, except when in Whiteboard mode

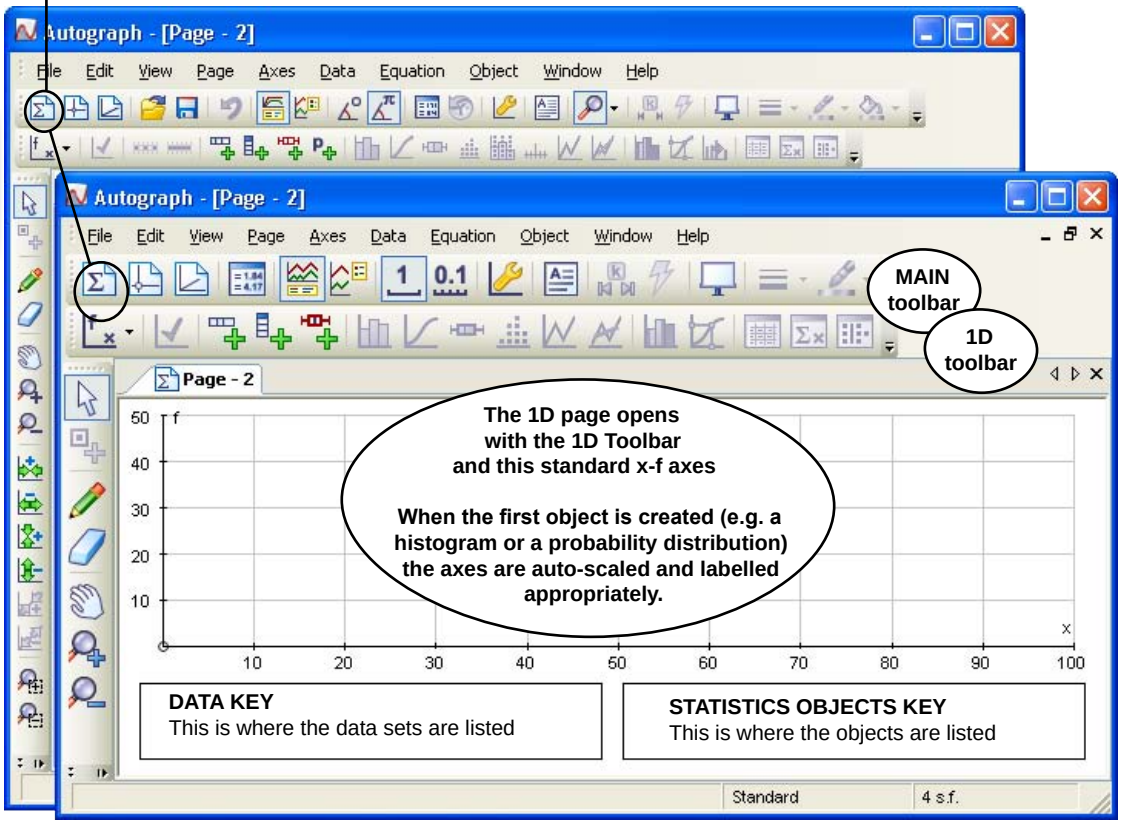
The MODES toolbar: get in the habit of returning to the SELECT mode.
- Either click on or press ESC. Pressing ESC again also deselects any outstanding object selections.

A couple of useful features using the CTRL key:
- 2D Point mode: + CTRL enables a point to be placed at an INTERSECTION, either f(x) = 0 or f(x) = g(x)
- 3D DRAG mode: + CTRL zooms the camera IN and OUT as you move the mouse UP and DOWN.

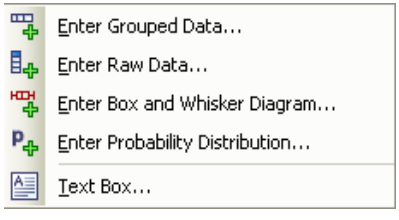
1D PAGE: Statistics and Probability



1D: Standard and Advanced



THE 1D PAGE:
The TOP LEVEL
RIGHT CLICK MENU



- **Enter Grouped Data**
  Set up the class intervals for a grouped data set. Frequencies are either entered manually, or calculated from underlying raw data.
- **Enter Raw Data**
  Enter or paste raw data, or generate it from one of Autograph's built-in probability distributions.
- **Enter Box and Whisker Diagram**
  You can create a box and whisker from its five elements: Minimum, Lower Quartile, Median, Upper Quartile, Maximum.
- **Enter Probability Distribution [Advanced Level only]**
  A selection of discrete and continuous distributions.
- **Text Box**
  Place a text box on the graphing area.

1D: Statistics Toolbar

THE 1D STATISTICS TOOLBAR

Re-label axes, frequency: f - x

Re-label axes, cumulative frequency: F - x

Re-label axes, continuous PDF: f(x) - x

Re-label axes, percentiles: % - x

Re-label axes, probability: p - r

These options offer a quick way to get the right axes labels, also possible through Edit Axes. New statistics objects automatically get the right labels and ranges when they are first drawn.

Auto-scale: use this when required.
This option will also re-label as appropriate.

Interpret current data set as discrete data

Interpret current data set as continuous data

These buttons are not on the Standard level, but you can set in the grouped data dialogue box.

Add **Grouped Data**

Add **Raw Data**

Add **Box and Whisker plot**

Add **Probability Distribution**
As featured on the top level Right-Click Menu

Draw **Histogram**
- from grouped data

Draw **Cumulative Frequency Diagram**
- from grouped data

Draw **Box and Whisker plot**
- from raw or grouped data

Draw **Dot Plot**
- from raw data

Sampling from raw data [Advanced level only]

Show **mean and 3 SD**
- puts pips on the 'x' axes

Draw **Line Plot**
- from discrete grouped data

Draw **Moving Average**
- from a Line Plot

Histogram **area calculation**

Cumulative Frequency **calculation**

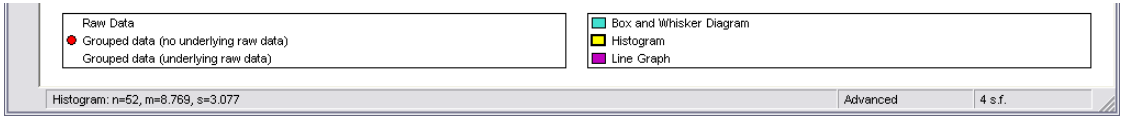
Probability distr. calculation [Advanced only]

Table of Statistics
- from grouped data

Comparison of Raw and Grouped **statistics**

Stem and Leaf Diagram
- from grouped raw data

THE DATA KEY (on the left) and OBJECTS KEY (on the right)



The DATA KEY lists all the datasets that may have been entered. If there is more than one data set, you can click on any one to make it the 'current' dataset, for which objects can be created. Each data set has a symbol to show what type it is (see below). To EDIT a data set, double-click on it in this list.

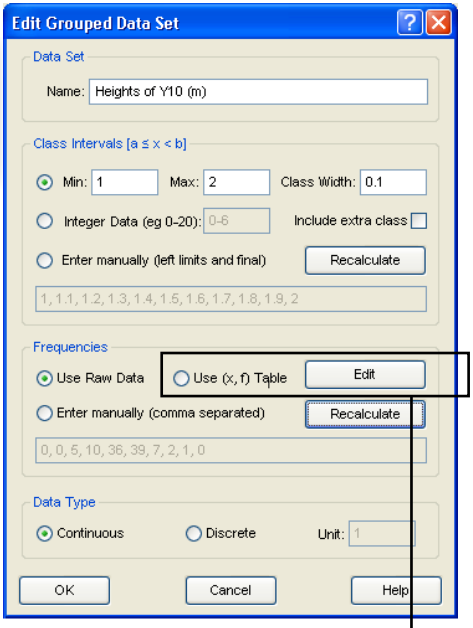
The OBJECTS KEY on the right lists the objects that have been created from the currently selected data set. Only one can be the current object, either by selection here or by clicking on the object itself in the graphing area.

KEY:

- Raw Data entered not grouped
- Grouped Data created from underlying raw data
- Grouped data entered no underlying raw data

1D: Adding a Grouped Data Set

The Edit Grouped Data Set dialogue box
- with or without an underlying raw data set.



DATA TYPE

♦ **Continuous:** classes will be plotted at their exact value.

♦ **Discrete, by entered unit (e.g. 1):** classes will be plotted shifted to the left by the unit/2. Required for a Line Plot.

GROUPED DATA SET

♦ **Name:** Enter the name as it will appear in the Data Set key.

CLASS INTERVALS - *choose one of the three options:*

♦ **Enter Min, Max and Class Width**
The convention adopted is **a ≤ x < b**. If the final value of the final class is needed tick "Include extra class". If there is underlying raw data, intelligent suggestions are available automatically.
The class width can be varied using the animation controller.

♦ **Integer Data**
Use this if the classes are all integers enter, e.g. Lotto frequencies (1-49), or dice scores (1-6). Equivalent to a class width = 1.

♦ **Enter manually (left limits and final)**
This is the chance to enter unequal class intervals.

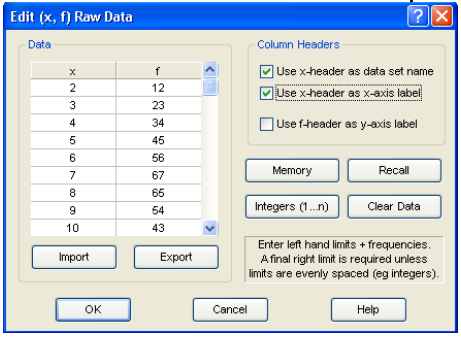
Recalculate
Click here to show the actual intervals that are being used.

FREQUENCIES - *choose one of the two options:*

♦ **Use Raw Data**
This will open the Edit Raw Data Set box, and the data from that will form the underlying data for this Grouped set. Frequencies will be calculated automatically according to the class intervals used.

♦ **Enter Manually (comma separated)**
Enter as many frequencies as there are classes.

Recalculate (with underlying Raw data only)
Click here to show the actual frequencies that are being calculated from the classes that have been entered (or amended).



♦ **USE (x,f) TABLE**

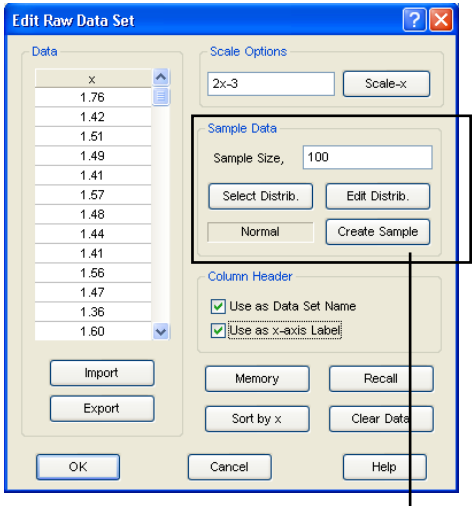
This will enter classes and frequencies in column form. The 'x' column represents the start of each group, and the (optional) final value is the end of the last group. These two columns can be pasted from a spreadsheet, and any text in the first row will become the column headers ("2-Dice scores" and "f" in the example).

The headers can be edited (right-click option), and used as the data set name and x-axis label .

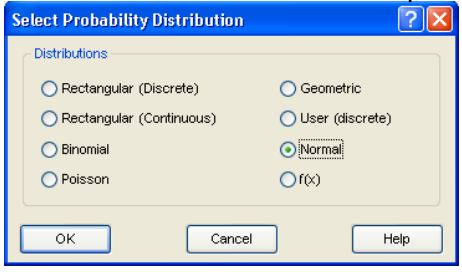
Memory/Recall: store and recall the current values
Integers (1...n): replaces the 'x' column with 1,2,3,...
This is useful, e.g., for Quarterly Data.
Import/Export: CSV file format.

1D: Adding a Raw Data Set

The Edit Raw Data Set dialogue box
- can be grouped or remain ungrouped.



DATA OUT
Hover over the column and use the right-click options: "Select All" (Ctrl-A), then "Copy" (Ctrl-C), then paste elsewhere.



SAMPLE DATA
Here you can create samples from any of Autograph's built-in probability distributions.

- **Sample size** (max 5000)
- **Select Distribution, then Edit Distribution**
- **Create sample**

The data will be added to any data already in the column.

RAW DATA SET

- Either: **Enter** the data one at a time, separated by "Enter".
- Or: Click on the top cell and **paste a single column** of data from a spreadsheet. If the top row is text, this will be taken as the "column header".
- Or: use **Import - Export**
A single column of data can be imported from a saved CSV file (comma-separated value), which is compatible with Excel.

SCALE OPTIONS
Enter any f(x) formula here. Click "Scale-x" to scale the data.

COLUMN HEADER
Here you can use the header as the Data Set Name, and the x-axis label when plotted. Hover over the column of data and use the right-click option to enter/edit the "column header".

MANAGING THE RAW DATA
Use "**Memory**", "**Recall**", "**Clear Data**" and "**Sort by x**" as required.

To Group a Raw Data set
You can reach this "Edit Raw Data" box as part of "Edit Group Data", or you can later use the right-click option "Group Data Set".

DISCRETE DISTRIBUTIONS

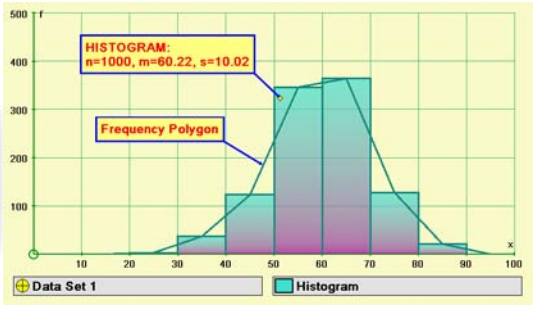
Select	*Edit*	*Default*	*Mean, Variance*
Rectangular	Enter a, b	r = 1, 2 ... 6	$\mu = (a + b)/2$ $\sigma^2 = (b–a)(b–a+2)/12$
Binomial	Enter n, p	B(10, 0.5)	$\mu = np$, $\sigma^2 = npq$
Poisson	Enter m	Poi(4)	$\mu = \sigma^2 = m$
Geometric	Enter p	G(0.5)	$\mu = 1/p$, $\sigma^2 = q/p^2$
User Defined	Enter p(0), p(1), ... p(r)		

CONTINUOUS DISTRIBUTIONS

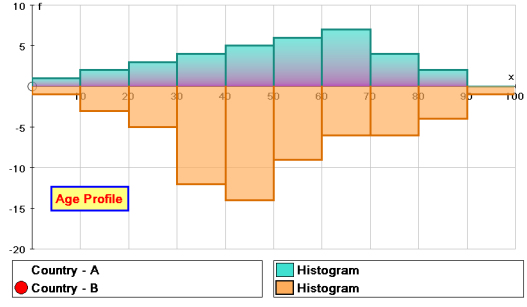
Select	*Edit*	*Default*	*Mean, Variance*
Rectangular	Enter a, b	$–2 \le x < 2$	$\mu = (a + b)/2$ $\sigma^2 = (a – b)^2/12$
Normal	Enter μ, σ	N(0, 1)	
f(x)	Enter f(x) and optional g(x), and ranges f(x) = x² range: $–2 \le x < 2$		

More than one sample can be 'mixed', e.g. to produce a bi-modal data set.

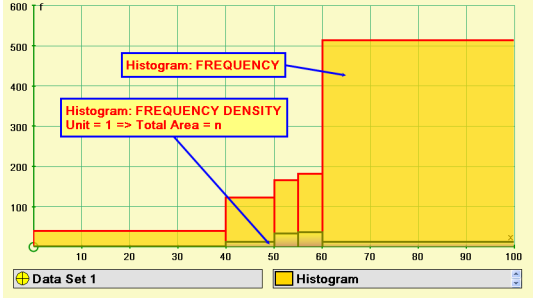
1D: Histogram



A histogram for a grouped data set (with underlying raw data), together with a Frequency Polygon. Use Edit Draw Options to control its appearance.



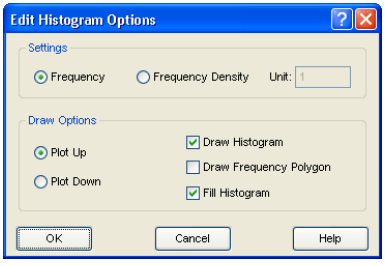
Two histograms back-to-back, one of them using the option "plot down". Use "Autoscale" option to create appropriate scales easily.



Two views of a data set with unequal class intervals: 0, 40, 50, 55, 60, 100. When the **frequency density** unit is set to '1' the histogram shrinks to an Area = n.

HISTOGRAM

A Histogram, plotting frequency or frequency density against 'x', can be created from any grouped data set (with or without underlying raw data).



♦ **Frequency Density**
Choose 'frequency density' and enter a 'unit'.
Unit = 1 ⇒ the total area under the histogram = n.
Unit = 1/n ⇒ the area = 1 (relative frequency diagram).

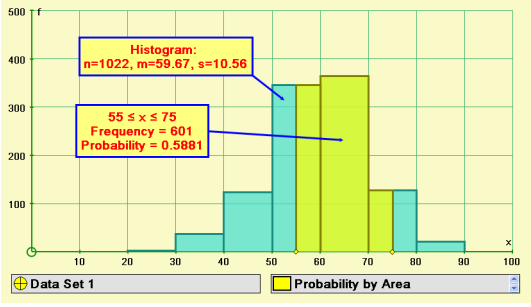
♦ **Plot up/down**
A good opportunity to compare two similar histograms. Use Autoscale to adjust the axes accordingly.

♦ **Draw Frequency Polygon**
This joins the upper mid-points of each class interval to form a polygon. To extend at both ends to zero it is necessary to have one 'spare' class at each end. Double-click on the data set name to return to the Edit Grouped Data box to extend the range of the classes.

Continuous Discrete

These two buttons are on the Advanced toolbar, but this choice is available in the Grouped Data dialogue box. In the 1D Statistics page, when histograms are plotted the 'x' scale is always continuous, so the following applies:

The convention adopted by *Autograph* is that the class interval a-b is represented by $a \le x < b$. When the variables are being plotted as discrete, of unit m (usually m = 1), the interval a-b is represented by $a-m/2 \le x < b-m/2$. With m=1, the effect of switching to discrete is to move the whole diagram to the left by 0.5.

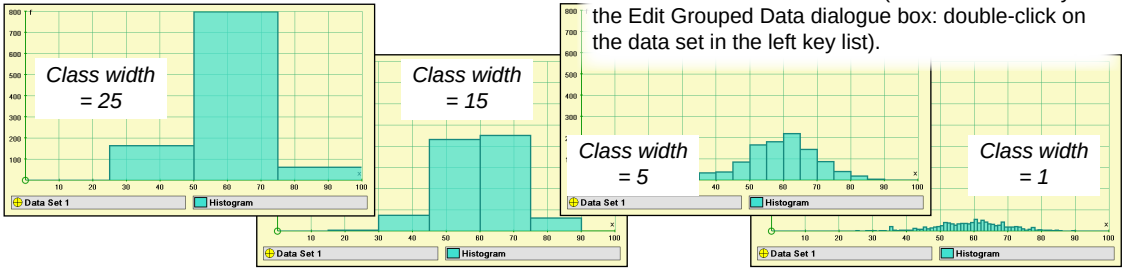


Histogram "Probability by Area" calculation.

To use the toolbar button or right-click option, make sure the histogram is selected in the right key list, or click on the histogram itself. In the dialogue box, enter range, or left or right cumulative limits. In the status bar, the probability and the frequency in the resulting region are given.

In this example: the probability and frequency for the range 55 ≤ x ≤ 75 are given. The yellow 'diamonds' can be dragged to vary the range dynamically.

If the data set is grouped with regular classes, the **class width** can be **animated** (or altered manually in the Edit Grouped Data dialogue box: double-click on the data set in the left key list).



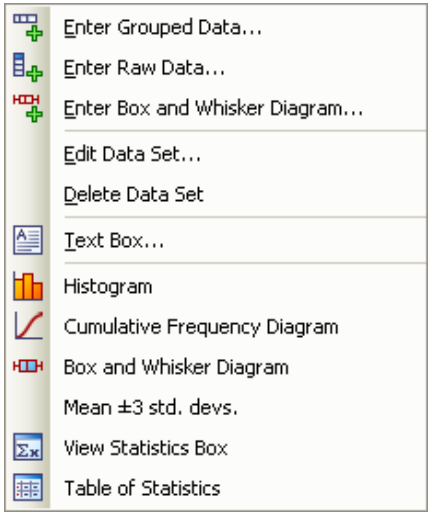
ROUTES TO ENTERING RAW DATA

If you know you want your raw data grouped (eg to draw a Histogram or Cumulative Frequency Diagram):

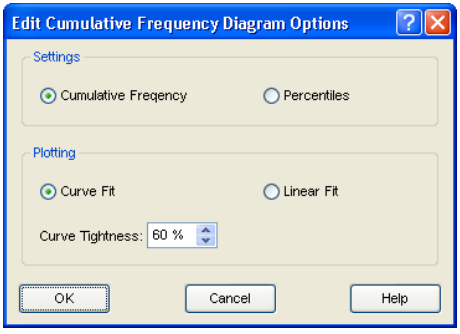
"**Enter Grouped Data**" - this dialogue box can create grouped data with underlying raw data (⊕) or without (●). It includes the option "**Use Raw Data**", which opens a dependent "Edit Raw Data" dialogue box, after which sensible default classes are offered.

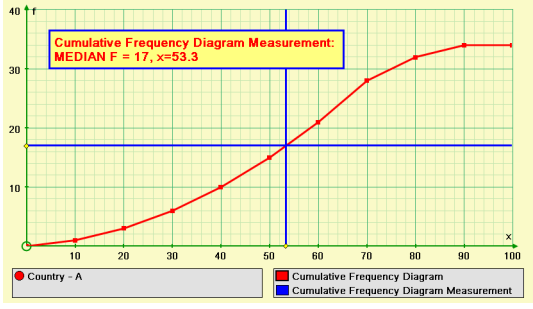
Otherwise, to enter raw data on its own (e.g. to draw a Dot Plot or Box and Whisker Diagram):

"**Enter Raw Data**" - this dialogue box allows a raw data set (⊕) to be created which is not grouped. To group this data subsequently, use the right-click option "**Group data set**".



1D: CFD and Box Plots





CUMULATIVE FREQUENCY DIAGRAM

The CFD option is available for any grouped data (with or without underlying raw data).

♦ Settings

Cumulative Frequency – uses the frequency scale of the vertical axis
Percentiles – converts so that max. frequency = 100%

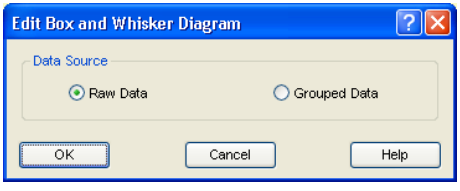
Plotting :
Curve fit: fits quadratics through three points at a time
Curve Tightness: 0% is the loosest, 100% is straight line segments
Linear Fit: straight line segments

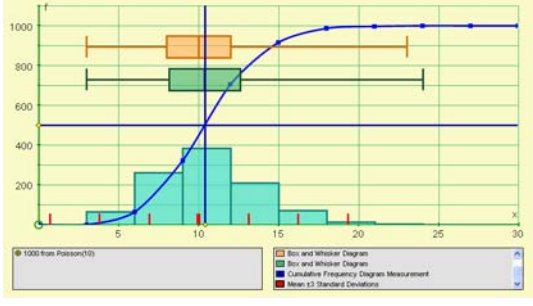
CFD MEASUREMENT

♦ Settings

Choose the **horizontal line**: LQ (25%), Median (50%), UQ (75%), or User Defined (enter value of 'F').
Choose the **vertical line** (default is near the minimum 'x' value).

Once drawn, both lines have a small yellow diamond which can be dragged as required to intersect on the CFD.





BOX AND WHISKER DIAGRAM

Data Source

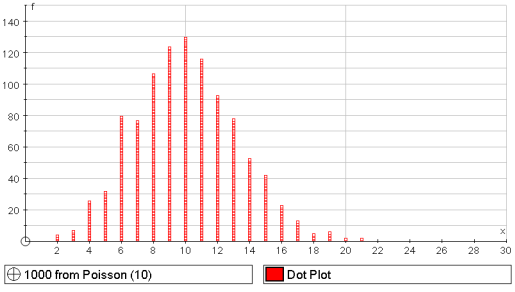
This diagram is constructed from the Lower and Upper Quartiles which form the 'box', the Median which is indicated by a vertical line in the box and end points of the whiskers indicate the range of the data.

The choice is to base it on the groups of the grouped data, or (if present) the raw data.

Raw data calculations:
The **LQ** is the (n+1)/4th value in ascending order.
The **median** is the 1(n+1)/2th value in ascending order.
The **UQ** is the 3(n+1)/4th value in ascending order.

Grouped data calculations:
the quartile and median are worked out by linear inter-polation. The resulting boundaries will correspond with the results found off the CFD only if the linear drawing options has been selected in the CFD dialogue box.

1D: Dot Plot and Line Plot

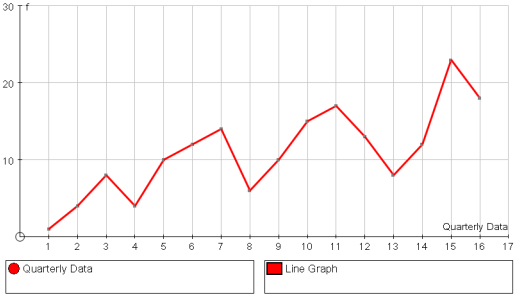


DOT PLOT

A dot plot can be drawn from any raw data set (whether grouped or not). The raw data will be displayed according to the settings in the dialogue box:

♦ **Horizontal spacing:** this effectively groups the raw data for the purposes of positioning the dots, e.g. setting this = 1 will display discrete integer data.

♦ **Vertical spacing:** this determines a link with the vertical axis so the stacking can be controlled.

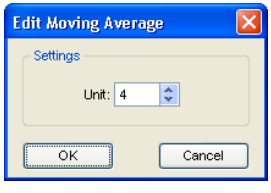


LINE GRAPH

A line graph can be drawn from any discrete grouped data set, with or without underlying raw data.

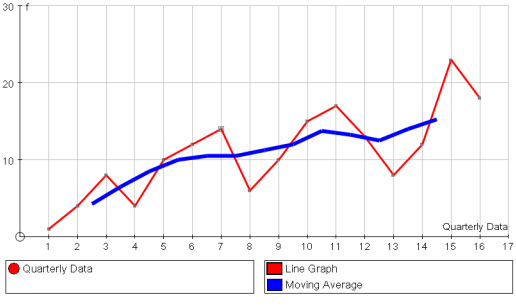
Data can be set as 'discrete' in the Edit Grouped Data dialogue box

Suitable data can also be created with the "**Use (x,f) Table**" option, especially if it exists in column form. The option "Integers 1...n" is particularly useful, eg for Quarterly data as this creates a set of integers for the x values. This is no different from entering the frequencies as a comma separated list and using the option "Integer Data" for the groupings.



MOVING AVERAGE

With a Line Graph already in place, a moving average can be drawn to a given unit (the default is 4, which suits quarterly data). Each of the points making up the moving average is placed in the middle of the group unlike Excel which places the points at the end of each group.

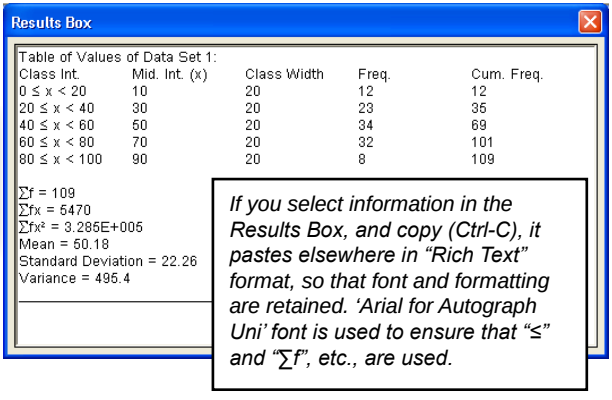


MOVING THE POINTS ABOUT

If you select any single point on the Line Graph, you can move it up and down with the UP/DOWN arrow keys, and observe the effect on the moving average. Use the LEFT/RIGHT arrows to select different points. This activity does not alter the data set.

The Moving Average 'Unit' can be animated.

1D: Table of Statistics



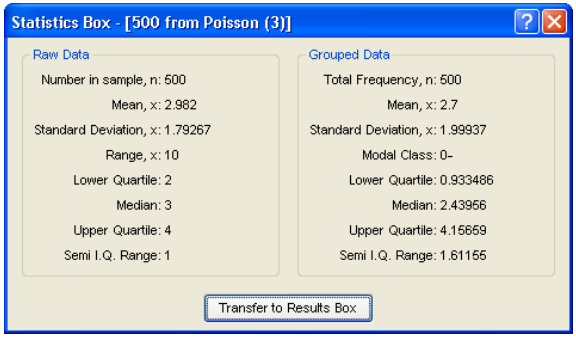
If you select information in the Results Box, and copy (Ctrl-C), it pastes elsewhere in "Rich Text" format, so that font and formatting are retained. 'Arial for Autograph Uni' font is used to ensure that "≤" and "∑f", etc., are used.

TABLE OF STATISTICS

This Table option is available for any grouped data set (whether with underlying raw data or not). The information appears in TAB-separated columns in the Results Box, and the class intervals are represented differently for continuous and discrete data:

Continuous data
The class intervals are $0 \le x < 20$, $20 \le x < 40$, *etc.*

Discrete Data
The class intervals are $0 \le x \le 19$, $20 \le x \le 39$, *etc.*

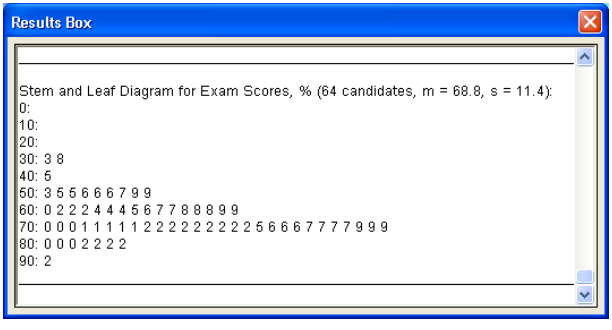


STATISTICS BOX

This option is available for all types of data, and offers a useful comparison between Raw data and Grouped data calculations.

Grouped data calculations are based on the mid-interval values. Linear interpolation is used to calculate the median and quartiles.

Raw data calculations use an ordered list of the actual data.

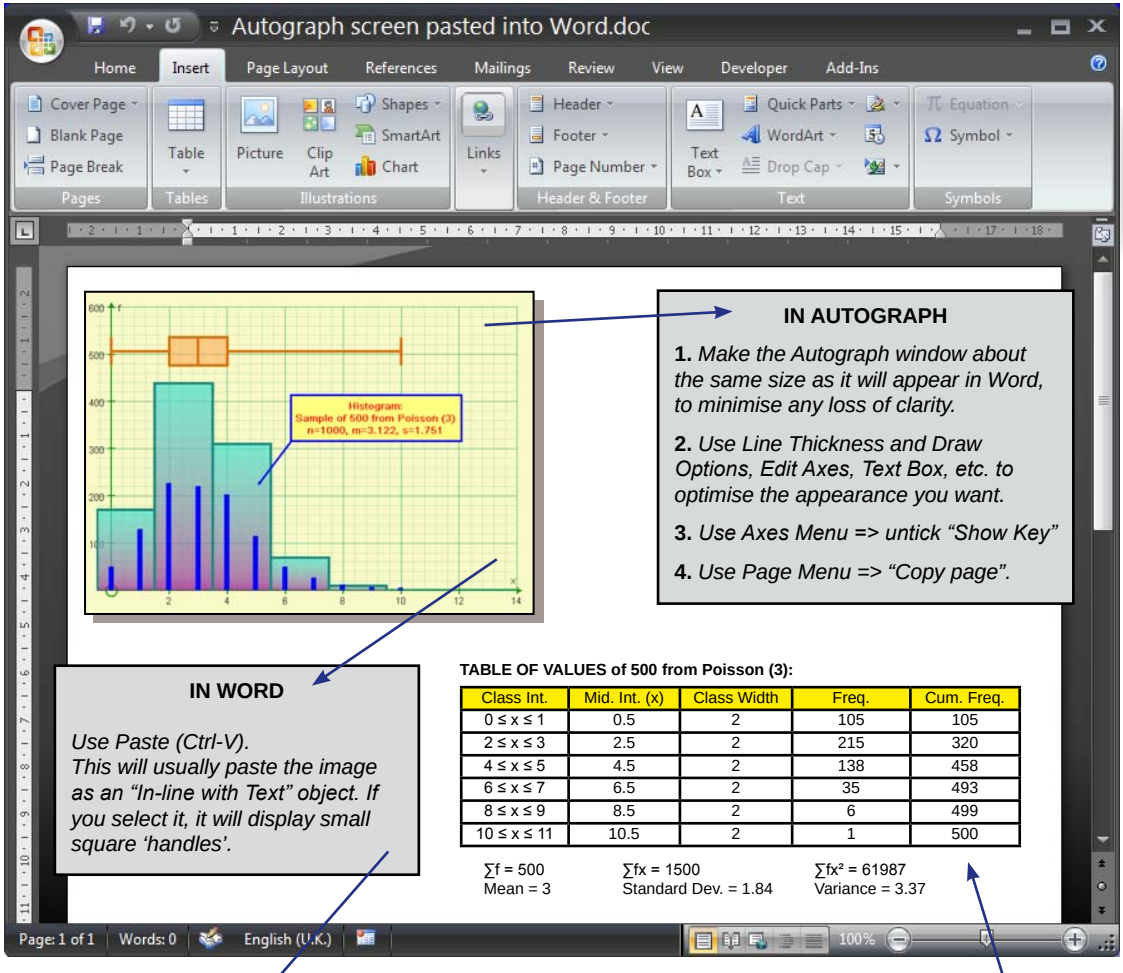


STEM AND LEAF DIAGRAM

For a grouped data set this diagram is constructed in the results box. Usual conventions are followed: the 'Stem' uses the LEFT end values of each class interval. The 'Leaves' use each data value belonging to each stem, and the difference between the data value and the stem is given.

This diagram really only works if the data is discrete, and best when the data is integer valued. Since it is created in the results box using text, the stem and leaf diagram can be selected, copied (Ctrl-C) and pasted (Ctrl-V) as required.

1D: Exporting to 'Word'



IN AUTOGRAPH

1. *Make the Autograph window about the same size as it will appear in Word, to minimise any loss of clarity.*

2. *Use Line Thickness and Draw Options, Edit Axes, Text Box, etc. to optimise the appearance you want.*

3. *Use Axes Menu => untick "Show Key"*

4. *Use Page Menu => "Copy page".*

IN WORD

Use Paste (Ctrl-V).
This will usually paste the image as an "In-line with Text" object. If you select it, it will display small square 'handles'.

TABLE OF VALUES of 500 from Poisson (3):

Class Int.	Mid. Int. (x)	Class Width	Freq.	Cum. Freq.
0 ≤ x ≤ 1	0.5	2	105	105
2 ≤ x ≤ 3	2.5	2	215	320
4 ≤ x ≤ 5	4.5	2	138	458
6 ≤ x ≤ 7	6.5	2	35	493
8 ≤ x ≤ 9	8.5	2	6	499
10 ≤ x ≤ 11	10.5	2	1	500

$\sum f = 500$ $\sum fx = 1500$ $\sum fx^2 = 61987$
Mean = 3 Standard Dev. = 1.84 Variance = 3.37

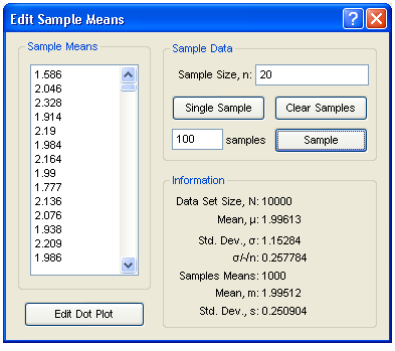
ADDING A BORDER AND DROP SHADOW

In WORD 2007, the contents of the old 'drawing Toolbar' are included in the "Insert" thread. First select the image, then insert a text box round it. Then select the text box object, and use the 'format text box' thread that appears, to add a shadow.

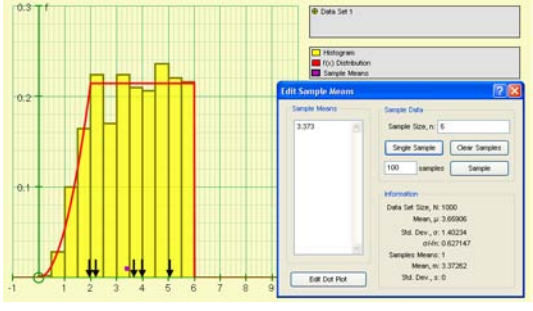
THE TABLE OF VALUES

Select and copy (Ctrl-V) the table in Autograph (in the Results Box). Paste onto the Word page. The mathematical symbols used (≤, ∑f, etc.) are in their standard Unicode positions, so any font may be used. Use Word's Table options to beautify!

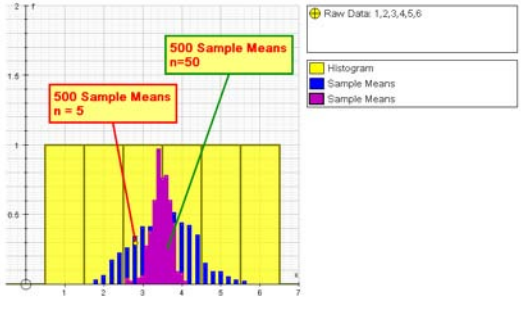
1D: Sampling and Sample Means



The dialogue box for setting up sample means, and thus testing the Central Limit Theorem.



A single sample of size 5 from a composite continuous function, indicated by black arrows, and the mean. The parent population was 1000 samples, displayed as a Histogram with frequency density unit = 1/1000.



A simple data set of the numbers 1,2,3,4,5,6, and the distribution of 500 samples of size n = 5 and n = 50.

SAMPLING FROM RAW DATA

You can create samples of size 'n' from any raw data (whether grouped or not). *[Advanced Level only.]*

A good way to create a large population is in the "Edit Raw Data" dialogue box, using the "Sample Data" option - there is a maximum of 10000 in any one sample.
The first example on the left is a sample of 10000 from a continuous pdf.

SAMPLE DATA

♦ **Sample size, 'n' (max 10000)**
Enter the desired sample size (minimum n = 1)

♦ **Single Sample**
Here the individual values of the sample are indicated by black arrows on the horizontal axis, together with the sample mean.

♦ **Clear Sample**
Clears all values, leaving the dialogue box open.

♦ **Sample**
Enter the number of samples (default = 100). This may be pressed many times. Each time the values of the sample means are displayed in the "Sample Means" column, and a dot plot is created on the graph.

SAMPLE MEANS

This column of data can be selected (Ctrl-A), copied (Ctrl-C) and pasted elsewhere (Ctrl-V).

EDIT DOT PLOT

You may well need to adjust the x- or y- spacing to suit the distribution being created.

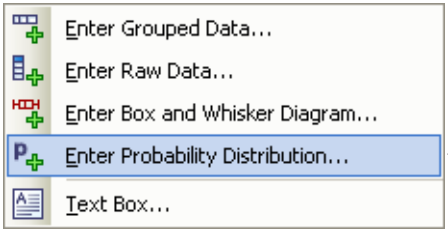
INFORMATION

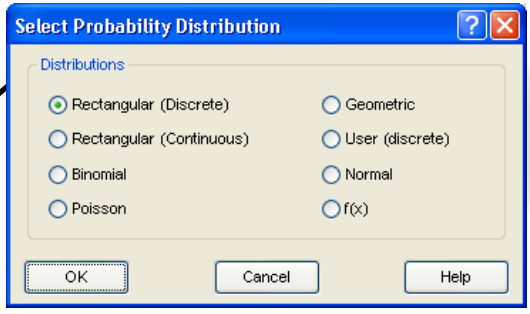
Evidence to support the **Central Limit Theorem** e.g.:

Data Set Size, N:	6	[Sample size, n = 50]
Mean, μ:	3.5	
Std. Deviation, σ/√n:	0.241523	
Sample Means:	500	
Mean, m:	3.47672	
Std. Dev., s:	0.226717	

1D: Probability Distributions

At the Advanced Level, the 1D page right-click menu includes the option to create probability distributions:





DISCRETE probability distributions
These are plotted in Autograph for integer values of 'r', usually starting with r = 0, using vertical bars. The sum of all the probabilities = 1, and the probabilities in any range or cumulative range of 'r' can be calculated.

CONTINUOUS probability distributions
These are plotted as a function of x. Probabilities are calculated by areas, and the total area = 1.

All discrete distributions can generate a **table of values**, which includes cumulative values.

The **animation controller** can animate any of the distribution parameters; any dependent objects will alter accordingly.

Binomial, Poisson and Normal can have their parameters chosen automatically to **fit data**.

The **Mean ± 3SD** can be added to any distribution, as small vertical ticks on the 'x' (or 'r') axis.

You can use "**Autoscale**" freely with any of the distributions. The first drawn is always rescaled and relabelled automatically.

You can always **edit the axes** manually.

All the distributions can have their **thickness** and **colour** altered.

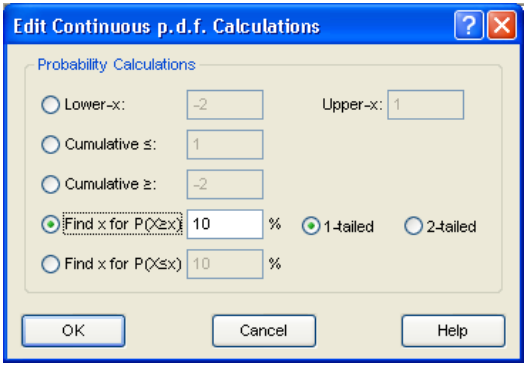
can be used to set up appropriate **labels**.

♦ PROBABILITY CALCULATIONS

All distributions can have a range probabilities calculated.

Discrete distributions:
Either enter Lower and Upper limits,
or choose to find a cumulative probability.

Continuous distributions:
These can also have inverse calculations for one- and two- tailed tests. e.g.: 5% 1-tailed, or 5% 2-tailed (=> 2.5% at each end).



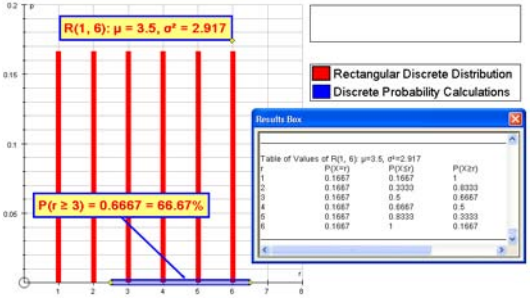
1D: Discrete Probability Distributions

RECTANGULAR (Discrete)

♦ Enter the start and finish values of 'r' (between 0 and 10000) to create the uniform discrete distribution.

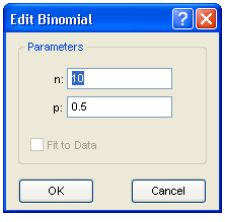
Once entered, you can measure probabilities, and create a table of values (in the Results Box).

Using the "Key right" button creates room on the right side to display other windows, e.g. the Table of Statistics.



A rectangular discrete distribution (six equal probabilities of 1/6) with a simple calculation and the Table of Statistics displayed in the Results Box.

BINOMIAL DISTRIBUTION



♦ Enter 'n' (an integer less than 100000) and 'p' (0 ≤ p ≤ 1).

♦ **Fit to Data:** with data entered, you can try to fit a Binomial with 'n' and 'p' such that its mean = np and variance = npq. If so, *the plot is scaled up by a factor of 'n'.*

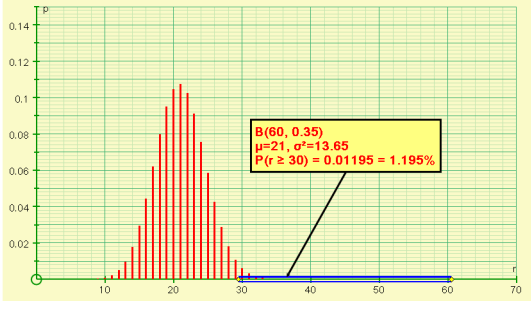
The animation controller can be used to animate 'n' or 'p', which will alter the binomial and any dependent calculations or objects (Normal or Poisson).

TABLE OF STATISTICS

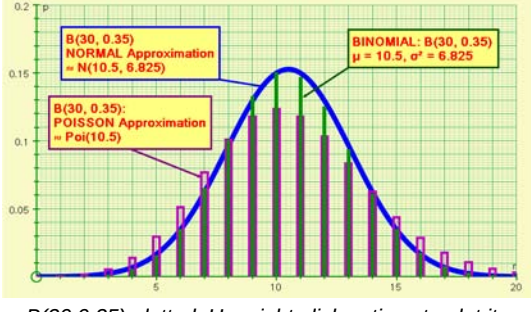
r	B(30, 0.35)	N(10.5, 6.825)	Poi(10.5)
1	0.00003942	0.000220865	0.000289133
2	0.0003078	0.000812938	0.00151795
3	0.001547	0.00258864	0.00531281
4	0.005621	0.00713151	0.0139461
5	0.01574	0.016998	0.0292869
6	0.03531	0.0350535	0.051252
7	0.0652	0.0625443	0.0768781
8	0.1009	0.0965549	0.100902
9	0.1328	0.128972	0.11772
...	...	...	...
18	0.003056	0.00258864	0.0103508
19	0.001039	0.000812938	0.00572018

Probabilities < 0.001

The Table of Statistics: Columns 3 and 4 are either the cumulative values (≤ r and ≥ r), or, when there are dependent objects, the equivalent probabilities.



Binomial with n=60, p=0.35 plotted with a cumulative calculation, and 'r' scale chosen to include all the range 0-60, and hence the entire distribution.



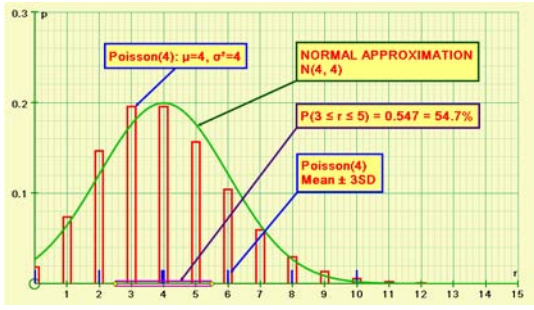
B(30,0.35) plotted: Use right-click options to plot its dependent Normal (μ = np, σ² = npq) and Poisson (λ = npq). Table of Statistics lists all three in columns.

POISSON DISTRIBUTION

♦ Enter the mean (between 0 and 10000) to create the distribution.

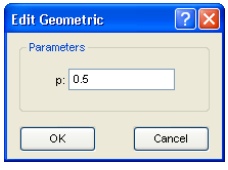
♦ **Fit to Data:** with data entered, you can fit a Poisson with its mean = data mean, and *the plot is scaled up by a factor of 'n'.*

Using the "Text Box" button you can add information onto the page from the Status Bar. Status Bar text can also be copied in the "Page" menu.



The Poisson distribution always has "twin towers" when its mean is an integer. Here the dependent Normal (μ = m, σ² = m) is drawn, and a probability calculated.

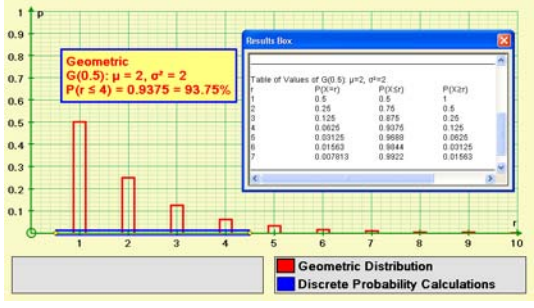
GEOMETRIC DISTRIBUTION



♦ Enter 'p' (0 ≤ p ≤ 1) to create the Geometric distribution.

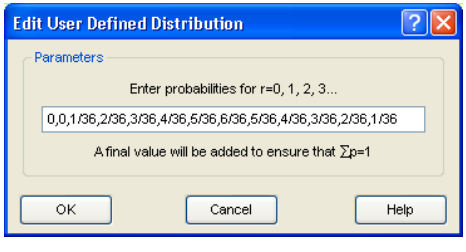
♦ Once entered, you can measure probabilities, and create a table of values (in the Results Box).

Using the "Autoscale" button, you can rescale the axes automatically at any time - this is often useful if the animation controller is in use.

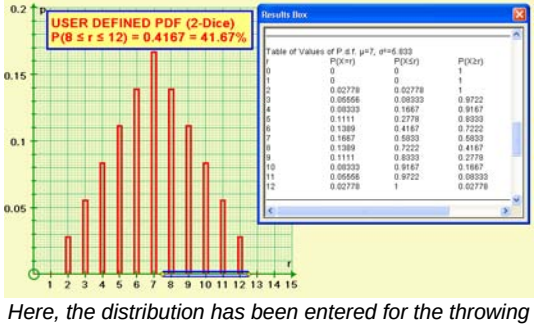


The Geometric shows the probability of the number of Binomial events until there is a success. It is therefore always a decreasing distribution.

USER DEFINED DISCRETE PDF



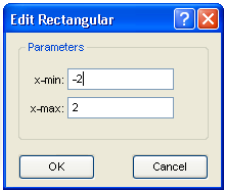
For any discrete pdf enter the probabilities for r = 0, 1, 2, etc, separated by ','. A final value is added in to ensure that ∑p = 1.



Here, the distribution has been entered for the throwing of 2-dice, and the probability for r > 7 shown, together with the Table of Statistics (with cumulative values).

1D: Continuous Probability Distributions

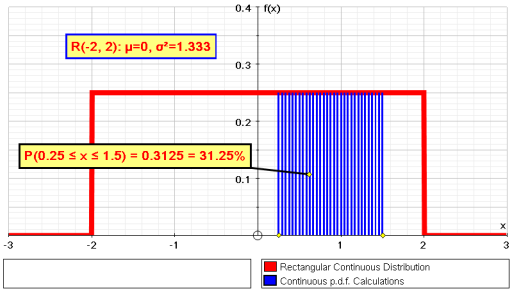
UNIFORM, Rectangular (Continuous)



♦ Enter the start and finish values of 'x' (Default is -2 to 2) to create a uniform distribution with Area = 1.

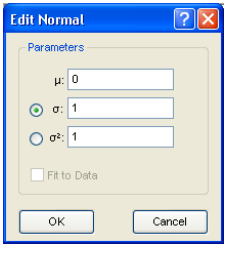
Once entered, you can measure probabilities, which are displayed as a shaded area under the distribution.

For all continuous functions, measurement options include inverse calculations and one- and two-tailed testing.



The uniform distribution can have any start and finish values; here a probability calculation is being carried out, and the results displayed in a text box.

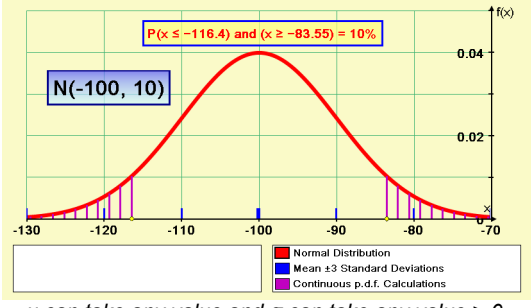
NORMAL DISTRIBUTION



♦ Enter mean and either SD or Variance.

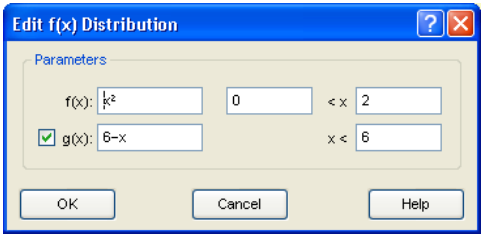
♦ **Fit to Data:** with data entered, you can fit the normal with the same mean and variance as the data, and *the plot is scaled up by a factor of 'n'.*

The animation controller can be used to animate 'μ' or 'σ', and hence to illustrate the transformation from N(μ, σ²) to N(0, 1).

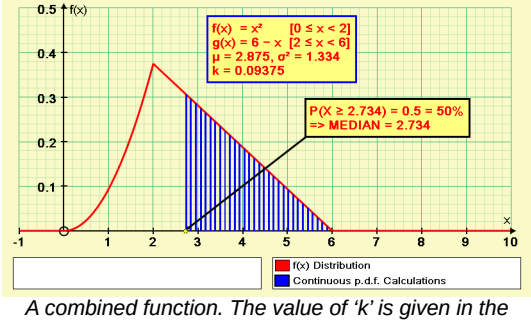


μ can take any value and σ can take any value > 0. More than one Normal distribution can be displayed at a time. Here a two-tailed test is under way.

USER DEFINED (CONTINUOUS)

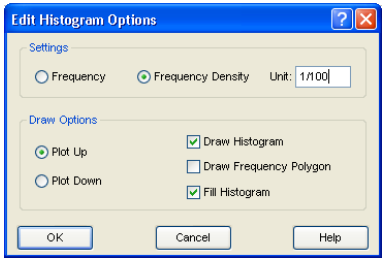


Enter any f(x) and its domain, then optionally a second g(x) up to a max value of x. The resulting function is scaled by 'k' to ensure the area is 1.

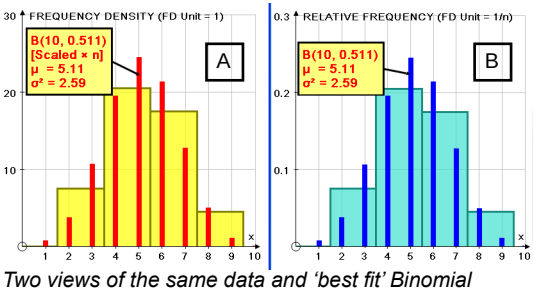


A combined function. The value of 'k' is given in the status bar. Here the inverse one-tailed calculation (50%) yields the median for this distribution.

FITTING A HISTOGRAM TO A PDF



Setting the Histogram Frequency Density Unit:
= 1 ensures the total area = 'n'
= 1/n ensures the total area = 1 [here n = 100]



Two views of the same data and 'best fit' Binomial
A: Histogram set to Frequency Density, Unit = 1
+ Binomial "Fit to Data" ticked in the Binomial entry.
B: Histogram set to Frequency Density, Unit = 1/n
+ Binomial entered: right-click option "Fit to data".

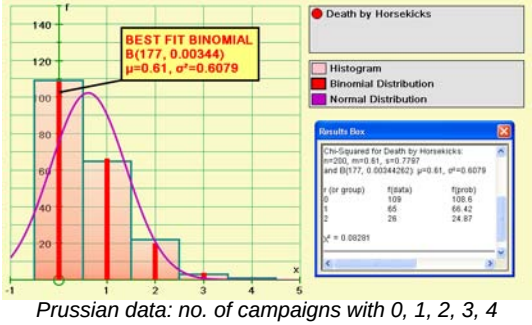
FITTING A BINOMIAL, POISSON or NORMAL

With a grouped data set plotted as a histogram, these three PDFs have a "Fit to Data" option on entry (see previous pages). The PDFs use best fit parameters, and are scaled up by 'n' (Σf). They are dependent objects (will change with the data).

Double-click on the Histogram to re-edit with Frequency Density set with unit = 1. It will shrink down to have total area = n, and to fit the PDF.

Use "Autoscale" as appropriate.

CHI With both Histogram and PDFs now displaying frequency, the χ^2 value is shown in Results Box.



Prussian data: no. of campaigns with 0, 1, 2, 3, 4 deaths by horse kicks: 109, 65, 22, 3, 1. Binomial (large 'n',small 'p': perfect fit) and Normal (poor fit).

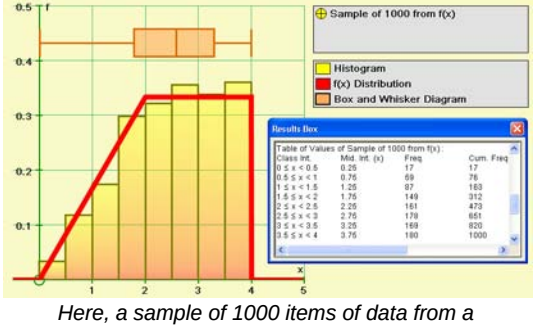
FITTING ANY Probability Density Function

With a grouped data set plotted as a histogram, any of Autograph's PDFs can be entered as usual, but only the three above have the "Fit to Data" option, when there is no scaling by 'n'.

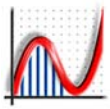
Double-click on the Histogram to re-edit with Frequency Density set with unit = 1/n. It will shrink down to have total area = 1, to fit the PDF.

Use "Autoscale" as appropriate

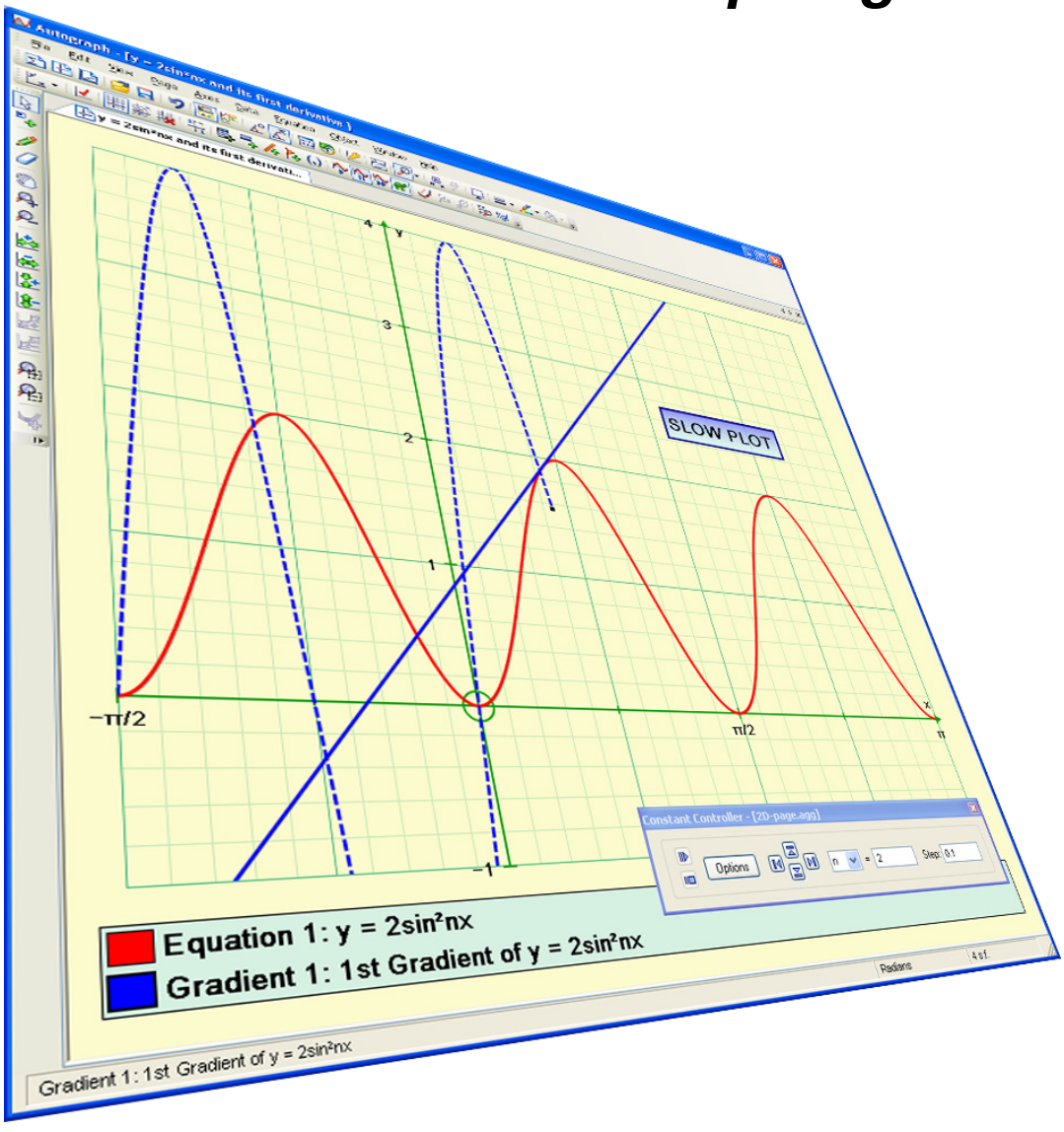
Use probability calculations as appropriate.



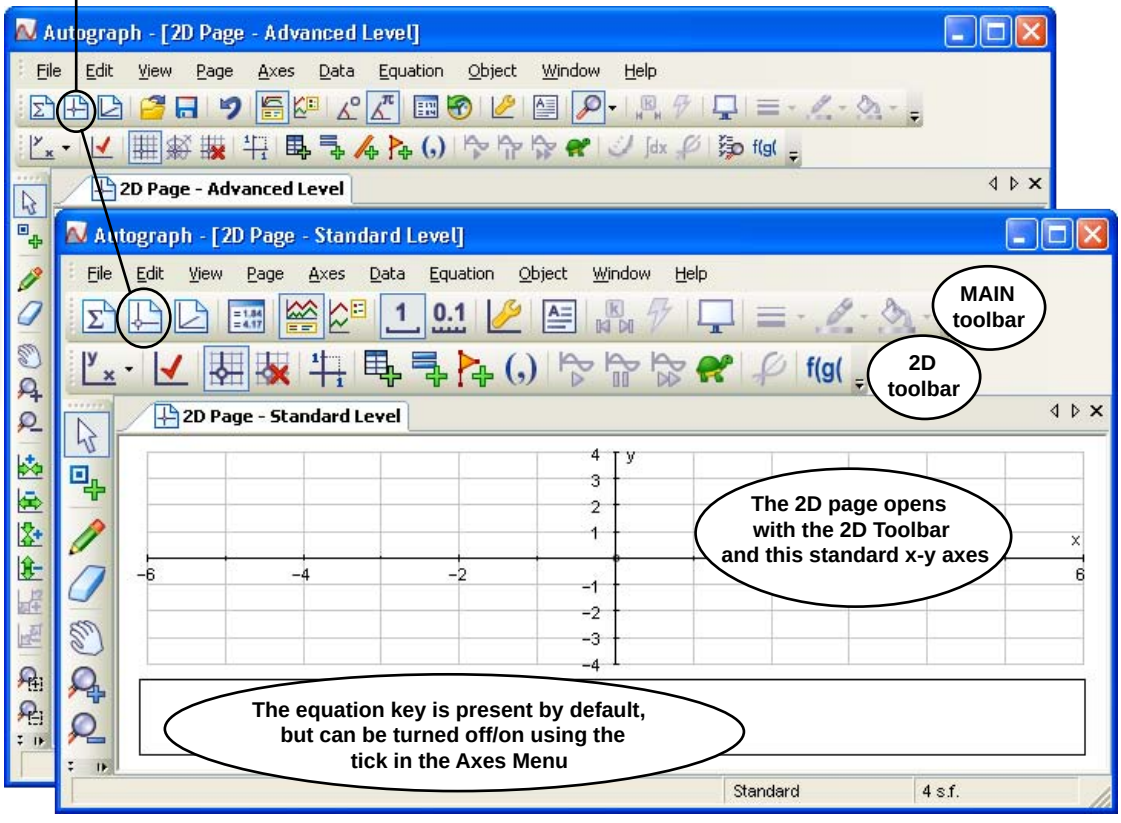
Here, a sample of 1000 items of data from a user-defined PDF (generated in "Edit Raw Data") is compared with the original PDF.



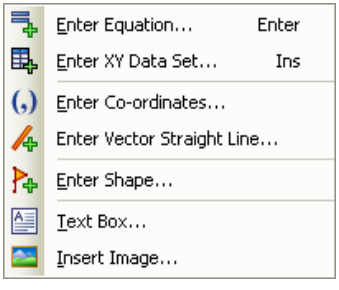
2D PAGE: XY Graphing



2D: Standard and Advanced



THE 2D PAGE:
The TOP LEVEL
RIGHT CLICK MENU



- **Enter Equation** [Pressing ENTER also opens this]
  The type of equation is detected automatically. *Constants can be included; Start-up options will depend on the type of equation.*
- **Enter XY Data Set**
  Enter or paste bivariate data. Axes can be swapped; data scaled.
- **Enter Co-ordinates**
  You can use numbers, expressions or constants.
- **Enter Vector Line [Advanced Level only]**
  A line in the form: [x,y] = [a, b] + λ[c, d]
- **Enter Shape:**
  A choice of 'preset' shapes, or enter a set of coordinates.
- **Text Box:** *Place a text box on the graphing area.*
- **Insert Image:** Place an image file on the graph page

2D: Graphing Toolbar

THE 2D GRAPHING PAGE TOOLBAR

AXES RELATED CONTROLS

Reset axes: y-x The default XY setting

Reset axes: x-t This is useful when 't' is the independent variable (e.g. differential equations)

Reset axes: x This allows work on the number line, with no 'y' axis

These options all re-start the page as they are likely to imply a change of variables. To avoid a reset use Edit Axes ("Labels" and "Options").

Default axes. This intelligently offers axes ranges appropriate to what is plotted: eg

y = sinx (degrees) x: –90° to 360°, pips 45°
y = sinx (radians) x: –π/2 to 2π, pips π/4
y = $\sin^{-1}$x (radians) y: –π/2 to 2π, pips π/4

and an XY data set has all points showing.

Cartesian grid The default

Polar grid [Advanced Level only]

No axes Ideal for some geometrical work

Equal Aspect - alters the x-scale so that the axes are square. Stays in force for the **current page**, whatever happens to the window, but is cancelled by any zoom action that contradicts it.

ADDING NEW TOP LEVEL OBJECTS

Add XY Data set

Add Equation

Add Vector Equation, Straight Line

Add Shape

Add coordinates

PLOTTING CONTROLS for classroom use

(*) These refer to the current equation (as indicated in the key), so you can pause one, restart another, etc.

Replot (*) - this will open the **Start up Options** (and option to change them) then replot.

Pause/restart (*) - in slow plot mode this controls the current equation (also SPACEBAR).

Fast Forward/complete (*) - in slow plot mode this will finish off the current equation.

Plot mode slow/fast. This stays on all the time, even with a new page. Strongly recommended.

CREATING RELATED GRAPHS

based on the current equation (as indicated in the key)

Create Gradient Function [Advanced only]
Drawn as a dotted line. When "Slow Plot" is on, this is an animation with a moving tangent, pausing at each max/min/point of inflexion.

Create Integral Function [Advanced only]
This uses the same routines as 1st Order Differential Equations; requires a starting point.

Create Reflection in y=x
Draws y=x, and the function with 'x' and 'y' swapped over.

Manage Equations
A look at the equation list, to copy/paste and turn off/on the list.

Define f(x) and g(x)
f(x) definitions can include g(x) and constants.

History - in the main toolbar, but useful here, for replotting previous equations.

USEFUL TOOLS from the MAIN toolbar

Key below and key right: to control layout

Degrees/Radians [Advanced Level only]

Constant controller; Animation controller

THE PAGE MENU

Use "Page" => "Edit Settings" to set the page name, angle measure and precision (sig. fig.) for the page. To set the precision permanently, use "View" => "Preferences". In the Advanced level, all new pages open in Radian measure.

2D: How to Enter and Edit Free Points

HOW TO ENTER FREE POINTS

Points can be '*free*', or '*attached*' to another object (e.g. a straight line or a graph), or can be *grouped* to form a SHAPE or an XY DATA SET. You can add points using the 'point mode' in the MODES toolbar. The last point is always left selected as the 'current' point and its coordinates are displayed in the Status Bar to the current decimal place setting (default: 4). **If you hold SHIFT, all the points are selected as you click to add them.**

The Standard level offers these buttons; the default 'Snap' setting = 1 for easy adding of integer points.

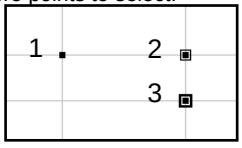
You can switch to a step of 0.1 using the other button, which is the default setting in the Advanced Level.

Alternatively, you can use this right-click option to "**Enter Coordinates**".
Coordinates can contain EXPRESSIONS, e.g. (π/3, 0),
or CONSTANTS, e.g. (a, b) which can subsequently be varied dynamically by the constant controller.
If you hold SHIFT as you place points, all the points are placed as selected points.

2D

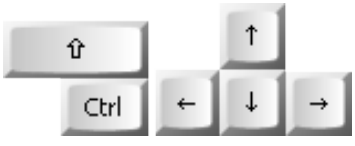
HOW TO SELECT POINTS

Select Mode: To ensure you are in Select Mode, either click the select button, or press 'ESC'.
Pressing ESC for the second time will also ensure that all previous selections are cancelled.
This is very important if you are in "No-Shift for multiple select" mode. Click on one or more points to select.



- There are THREE states for points in Autograph, whether they are 'free' or attached to curves: 1 - **UNSELECTED**, 2 - **SELECTED**, and 3 - the last to be selected and therefore the **CURRENT point**, whose coordinates are given in the Status Bar.
- You can move which is the 'current' point round the point set by pressing **TAB**.
- **MARQUEE SELECT:** In select mode, you can select points by dragging a rectangle round them with the mouse.
- Selected points can be **deleted** (press "Delete"). The next recent point then becomes the 'current point', so all points can be deleted one at a time by pressing "Delete".
- **TRACE:** With one or more points selected, the right-click option "Trace" will create a trace of the point(s) if any other action causes them to move.

HOW TO MOVE POINTS



A selected point can be moved about by dragging with the mouse, or by using the ARROW keys as follows:
LEFT-RIGHT will move it in the 'x' direction, UP-DOWN in the 'y'.

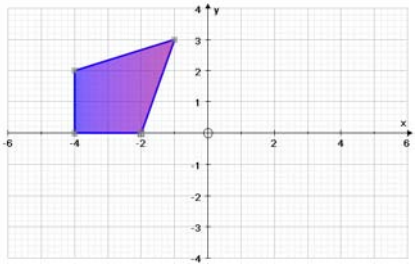
The default step is x 0.1, but holding CTRL: x 0.01; SHIFT AND CTRL: x 0.001.
Holding just SHIFT: x 10. All these steps are relative to the axes grid.

In the **Standard Level**, the 2D page starts with the 'Snap' set to 1, so all the above steps are multiplied by 10.

HOW TO EDIT POINTS

- **TO EDIT A POINT:** (as with all other objects in Autograph), double-click on it. Both coordinates can be edited.

2D: Other Uses for Points

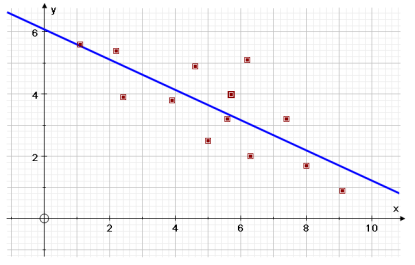


POINTS CAN MAKE A SHAPE

A selection of points can be made into a SHAPE using the right-click option “Group to Shape”.

To select the shape, click on its shaded area. A selected shape can be dragged about or moved using the ARROW KEYS (with Ctrl, Shift. etc, as with a single point).

A single point in a shape can be selected and dragged about with the mouse, or moved with the ARROW KEYS.

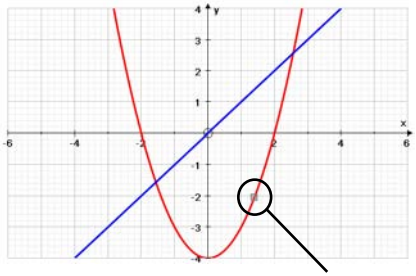


POINTS CAN MAKE AN XY DATA SET

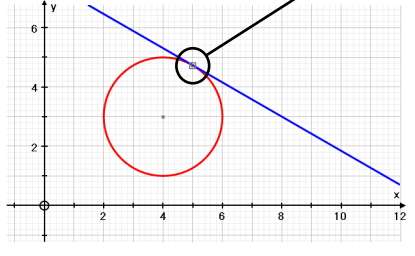
A selection of points can be made into an XY DATA SET using the right-click option “Create Data Set”.

To select an XY data set, click on any one of its points. To select an individual point in the dataset, use CTRL-click. With Ctrl, you can drag this point, or delete it.

You can use MARQUEE select to make a selection of several points in the data set. Use the right-click option on a selected dataset “Revert to points”.



Points can be attached to all objects, including graphs, lines, circles, etc



POINTS CAN BE ATTACHED TO OBJECTS

To attach a point to an object, in ‘Point’ mode, the mouse cursor will change to a **small black arrow** when you are over an object, such as a graph, straight line, circle, etc. This indicates that a left mouse-click will place and attach a point on the object.
[This is hard to do on a Smart Board as there is no ‘mouse’ signal].

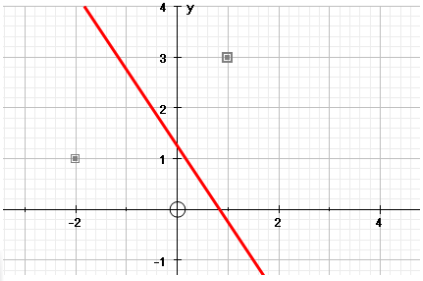
With many objects, e.g. a y=f(x) graph, you can also select the object and use the right-click option “**Enter point on Curve**”. You enter just one coordinate. This is usually ‘x’, but in the case of x = f(y) it is ‘y’, or a polar graph r = f(θ) it is ‘θ’.

With an attached point selected, you can **move the point along** the object by dragging with the mouse, or using the LEFT-RIGHT arrow keys.

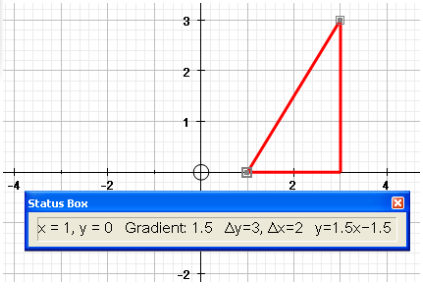
If there are two or more y = f(x) objects present, and a point attached to one, use the UP/DOWN arrow keys to **move the point between** the objects, with the same value of ‘x’.

With a free point and an object selected, use the right-click option “**Attach to object**” to move the point onto the object (same value of ‘x’).

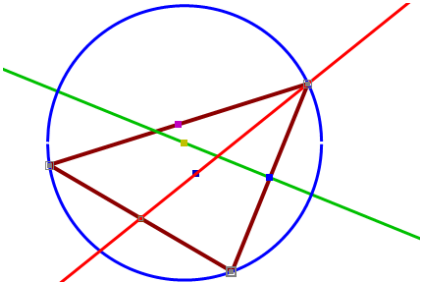
2D: The Straight Line



Perpendicular bisector



Gradient triangle and Status Bar information (in the Status Box)



3 points leading to a circle, a triangle, a perpendicular bisector (green) and an angle bisector (red)

THE STRAIGHT LINE

The straight line is an important object in Autograph, used for reflection and various intersections. For each, the equation is given in the **Status Bar** *in the form y = ax + b, or x = k. There are several ways to create lines, by these selections and using the stated Right-click option:*

ONE POINT selected:

- **Horizontal Line** y = a
- **Vertical Line** x = b
- **Straight Line** [enter gradient, m] y = mx + c

ONE POINT selected ON A GRAPH :

- **Tangent**
- **Normal**

TWO POINTS selected:

- **Straight Line** Equation given
- **Line Segment** Length AB given
- **Perpendicular Bisector** Equation given
- **Gradient** (right-angled triangle) Δy, Δx; equation of line; Δy/Δx

ONE POINT and a STRAIGHT LINE selected:

- **Parallel Line** Equation given
- **Perpendicular Line** Equation given

TWO LINES or LINE SEGMENTS selected:

- **Angle between the lines** Degrees or radians (Results Box)
- **Intersection point** Coordinates given (Results Box)

THREE POINTS selected (one is the 'current point'):

- **Angle bisector** (at current point) Equation given

y=f(x) GRAPH selected

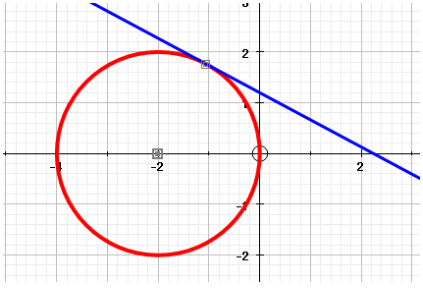
- **Asymptote** Tangent at x = ∞

XY DATA SET selected

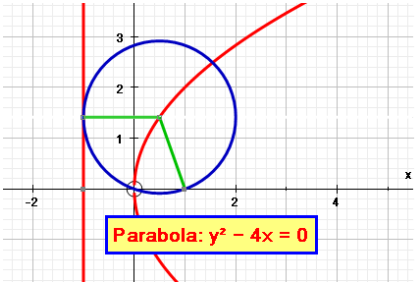
- **Line of best fit** [Standard Level]
- **y-on-x regression line**; **x-on-y regression line** [Advanced Level]

Also, you can ENTER the STRAIGHT LINE'S EQUATION:

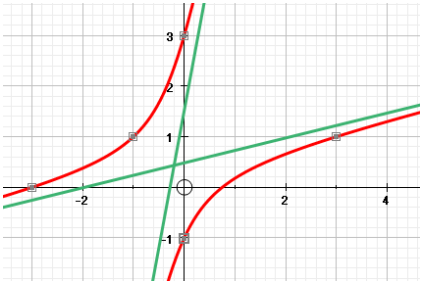
- **y = f(x) form:** y = mx + c
- **x = f(y) form:** x = k or x = my + c
- **Implicit form:** ax + by = c x/a + y/b = 1
  (y – b)/k = (x – a)/h
- **Vector form:** [Advanced Level] [x, y] = [a, b] + λ[h, k]



A circle from two points, and a tangent



Examples of a dynamic construction involving points, lines and circles



A conic constructed from 5 points. Each branch was selected separately and an asymptote drawn

All Cartesians conics constructed in Autograph are rearranged internally as y = ... ± √... , which have two branches. A point added can move round one branch (LEFT/RIGHT arrows), or can swap branches (UP/DOWN). Double-click to edit the point's 'x' coordinate.

THE CIRCLE

There are two distinct types of circles in Autograph, and for each form, the equation is given in the Status Bar as $(x - a)^2 + (y - b)^2 = r^2$.

CARTESIAN CIRCLES

THREE POINTS selected
♦ **Circle**

PARAMETRIC CIRCLES
A point added can be dragged right round the circle (LEFT/RIGHT arrows). Double-click to edit the point's parameter value and its 't' snap.

ONE POINT selected
♦ **Circle** [enter radius]

TWO POINTS selected (one is the 'current point')
♦ **Circle** Centre at the current point

ONE POINT and a LINE SEGMENT selected
♦ **Circle by centre and radius**

OTHER CONICS

The equations of all other constructed conics in Autograph are given in the form $ax^2 + by^2 + cxy + dx + ey + 1 = 0$.

THREE POINTS selected (one is the 'current point')
♦ **Ellipse** (two foci, and the current point on the perimeter)

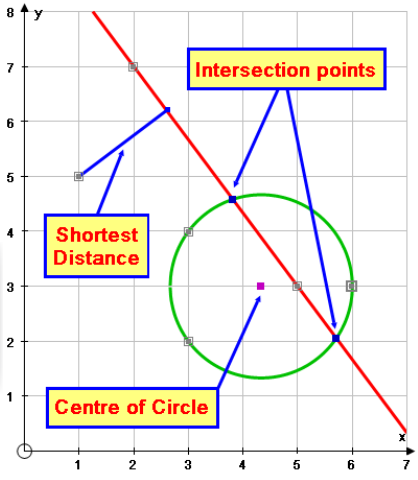
A POINT and a STRAIGHT LINE selected
♦ **Conic** (enter eccentricity)

FIVE POINTS selected
♦ **Conic**

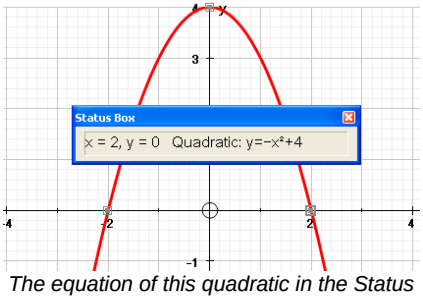
Also, you can ENTER the CONIC'S EQUATION

♦ **Cartesian:**	$x^2/a^2 + y^2/b^2 = 1$	$(x - a)^2 + (y - b)^2 = r^2$
	General conic:	$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
♦ **Polar:**	$r = k$	$1/r = 1 + k\cos\theta$ [k = 0, ½, 1, √2, 2]
♦ **Parametric:**	$x = at^2, y = 2at$	$x = a\sin\theta, y = b\cos\theta$

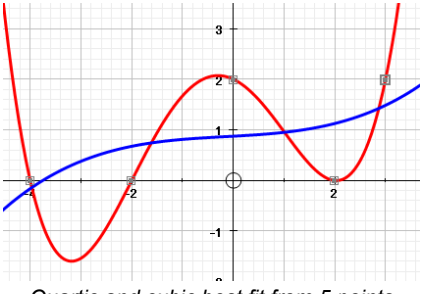
2D: Calculations



Examples of calculated points, and a shortest distance.



The equation of this quadratic in the Status Box (double-click on the Status Bar)



Quartic and cubic best fit from 5 points

CALCULATED POINTS

Autograph uses "Points" (those entered by the user), and "Calculated Points" - these are not added to the set of points, and are indicated by a small coloured square. They can be selected (for deletion), and a point can be attached to a calculated point.

TWO POINTS selected AB (where 'A' is the 'current' point)

- **Midpoint** — Coordinates (a, b) given
- **Ratio:** calculated point 'P' — Enter the ratio AP/AB

THREE POINTS selected

- **Centre of circle** — Coordinates (a, b) given

THREE or MORE POINTS, or an XY DATA SET selected

- **Centroid** — Coordinates ($\bar{x}$, $\bar{y}$) given

POINT 'P' and a CIRCLE (centre O, radius r) selected

- **Inversion point P'** — OP x OP' = r^2

TWO OBJECTS selected (e.g. 2 lines, 2 graphs, line and circle, etc)

- **Intersections** — List of points in the Results Box

*Alternatively, in POINT mode, hold CTRL and wait for the mouse cursor to change to a circle to place a **point** AT an individual intersection.*

A POINT and a STRAIGHT LINE selected

- **Closest point on the line** — Point marked; distance given

To place a point here, use point mode and click on the line.

CALCULATED BEST FIT CURVES

TWO POINTS selected (one is the 'current point')

- **Quadratic** (enter gradient) — $y = ax^2 + bx + c$

THREE POINTS selected (one is the 'current point')

- **Quadratic graph** — Equation: $y = ax^2 + bx + c$
- **Cubic graph** (enter gradient) — Equation: $y = ax^3 + bx^2 + cx + d$

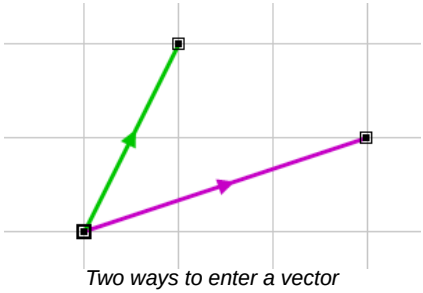
FOUR POINTS selected

- **Cubic Graph** — Equation: $y = ax^3 + bx^2 + cx + d$

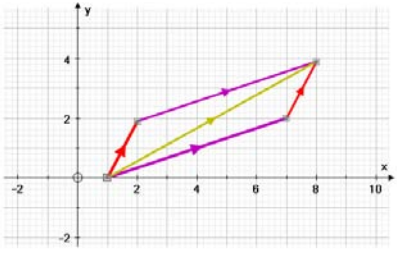
'n' POINTS or a DATA SET selected

- **Best Fit polynomial** — *up to degree 6*

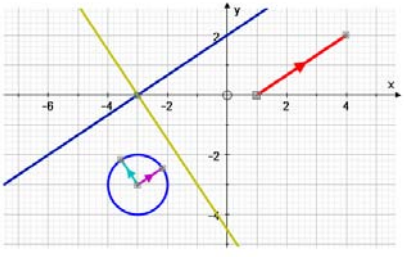
An exact polynomial of degree (n – 1) is given when only n points are selected up to n = 7. Otherwise a 'best fit' polynomial is given, just enter n (from 2 to 6). The method of least squares is used to derive it.



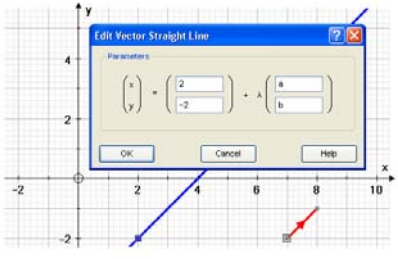
Two ways to enter a vector



Elementary vector algebra



Vectors and lines in 2D



The vector equation of a straight line, and a parallel vector, both using constants.

VECTOR CREATED FROM A SINGLE POINT
A vector must have a point to 'sit' on, so to create a vector you must have a point selected. Use the right-click option "Vector" and enter the coordinates, either Cartesian [x, y] or polar [r, θ]. If constants are used here, these can be altered using the Constant Controller.

The vector can then be moved by dragging the base point.

VECTOR CREATED FROM TWO POINTS
Alternatively, a vector can be created between two selected points, making it moveable at either end.

A vector consists of three objects: base point, end point and stem (with an arrow at the mid-point). To select a vector click on its stem.

2D

ONE VECTOR [a, b] and a POINT [h, k] selected,
a new vector can be created based on the point as follows:

- **Perpendicular vector** [–b, a]
- **Negative vector** [–a, –b]
- **Copy vector** [a, b]
- **Unit vector** [a/r, b/r], $r = \sqrt{(a^2 + b^2)}$
- **Multiply vector (enter factor)** [ka, kb]

This creates a scalar multiple of the original vector. 'k' can be animated.

- **Line from vector (*)** [x, y] = [h, k] + λ[a, b]
- **Parallel line (*)** y = mx + c
- **Perpendicular line (*)** y = mx + c

(*) These are all drawn through the selected point [h, k]

- **Closest point, and distance** *on the line of the vector*

TWO VECTORS [a, b] and [c, d] selected,
- **Angle between two vectors (*)** *in degrees or radians*
- **Scalar (or Dot) Product (*)** ac + bd

(*) *given in the Results Box*

TWO VECTORS [a, b] and [c, d] and a POINT [h, k] selected,
a new vector can be created based on the point as follows:

- **Add vectors** [a + c, b + d]
- **Subtract vectors** [a – c, b – d]

VECTOR EQUATION of a STRAIGHT LINE [Advanced level]

This can be added any time as it is a top level right-click option, using the form: [x, y] = [h, k] + λ[a, b] **You can use constants in any of the coordinates.**

2D: Adding an Image


In the right-click menu (or the 'Object' menu) use **Insert Image ...**

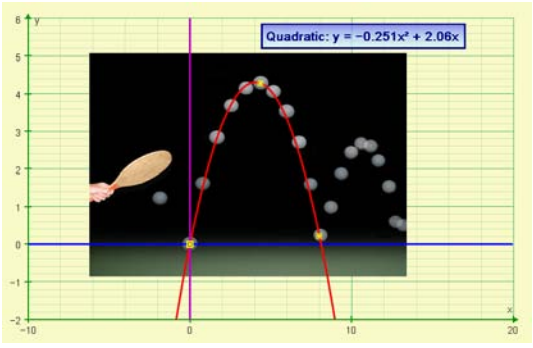
This opens a standard file window, and you can select files of all the popular types:
- emf [enhanced metafile], wmf [windows metafile], jpg, png, bmp, gif or tif

Initially the image is placed centrally and scaled to fit easily on the page. The image is an Autograph object.

- Click on it once to select it (for deletion or to drag it)
- Any of its 8 tags can be used to resize it dynamically.
- Dragging a corner will preserve the aspect

- Double-click on the image to edit its properties:
  Brightness, Contrast and Transparency - the effect is previewed instantly. These may need adjusting when it its planned to draw graphs over the image.

  Press "OK" to confirm, or
  press "Cancel" to return to the previous settings.



A stop-frame image of a bouncing ball, fitting a parabolic path

Scale Image with Axes:
Tick this to ensure that the image is locked to the axes and will scale with any zooms, etc. Otherwise it will stay its original size on the screen

2D: Measuring an Angle

Angles can be measured:

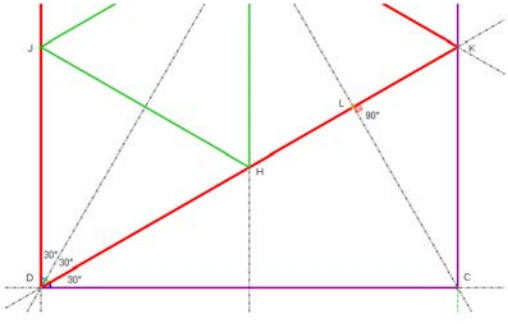
(a) defined by three selected points
    The angle is displayed at the middle point
(b) defined by two selected lines
    The angle is displayed at the intersection of the two lines
(c) Angle defined by two selected vectors
    The angle is displayed at the intersection of the two vectors

The angle 'object' is a filled circular arc.
If the angle is 90°, the usual rectangular symbol is used.

The appearance of the angle object can be altered using **Edit Draw Options** or by using the buttons on the main toolbar.

SETTINGS

Allow Reflex Angle - will show angles greater than 180°, measured *anticlockwise* from the first two points.
Angle Arc Size - stated in a nominal mm measurement.

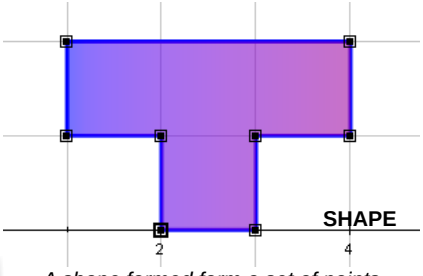


An example of a paper folding exercise simulated in Autograph, with key angles measured

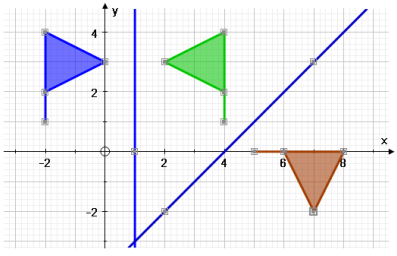
LABEL

Show Label - this will give the value of the angle
Edit Label Settings - this opens the dialogue box for a Dynamic Text Box, where you can set the appearance of the text, and whether it anchors to the angle object.

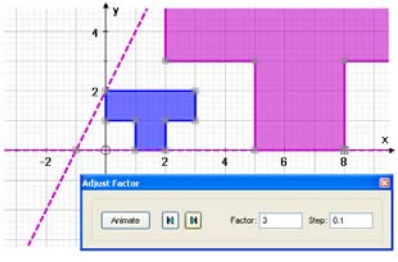
2D: Transformation of Shapes



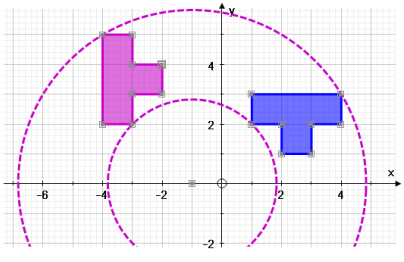
A shape formed form a set of points



Select a shape and a line for Reflection



Select a shape and a point for Enlargement



Select a shape and a point for Rotation

CREATING AND EDITING SHAPES

Shapes are created in Autograph by selecting a number of points and using the right-click option "**Group to shape**". Alternatively, use the "**Add Shape**" button to enter the coordinates, or use a pre-set shape (flag, unit square, rectangle). The default option is to **fill** the shape.

To select a shape, click on the shaded part. You can alter its appearance using the right-click option **"Edit Draw Options"**.

You can **drag** a shape about (dragging the shaded part), or you can drag any one of its points separately. To edit a shape, double-click on it. A selected shape can also be moved using the Arrow Keys.

CREATING DEPENDENT SHAPES

With a SHAPE selected, you can create a new, dependent shape:

- **a Reflection in the x- or the y-axis**
- **a Shear in the x- or the y-axis** (enter the shear factor)
- **a Stretch in the x- or the y-axis** (enter the stretch factor)
- **a Transformation** by entering a **2x2 matrix** [Advanced level only]

With a SHAPE and a POINT selected, you can create a new shape:

- **an Enlargement** (enter the initial factor)
- **a Rotation** (enter the initial angle)

With a SHAPE and any STRAIGHT LINE (constructed or by an equation) selected, you can create a new, dependent shape:

- **a Reflection**

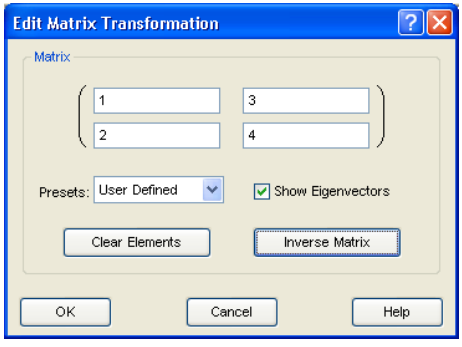
With a SHAPE and any VECTOR selected, you can create a second, dependent shape:

- **Translation**

If the vector is based on two points, you can animate this translation by moving the end point of the vector.

Animating a dependent transformed shape

To animate a shear, stretch, enlargement or rotation, select the new shape and use the animation controller to vary the factor or angle dynamically, or to set up an automatic animation.



First create and select a **shape**, and use the right-click option "**Matrix Transformation**" [Advanced level only].

To enter a **User Defined** matrix, enter each element, followed by TAB, noting that the order is by columns, to emphasize the transformation of the unit square defined by (1, 0) and (0, 1). The elements can include expressions (e.g. √3/2), or constants. Alternatively you can use one of the Presets (listed below).

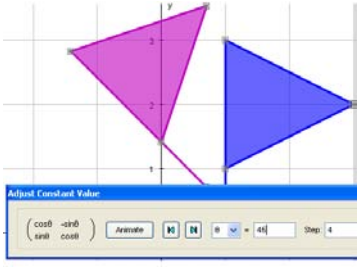
Show Eigenvectors: this option will draw the two eigenvectors as dotted lines (if they exist) and state their equations.

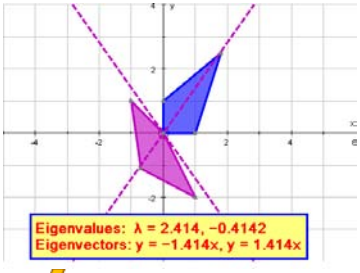
Inverse Matrix: this replaces the entered matrix with its inverse.

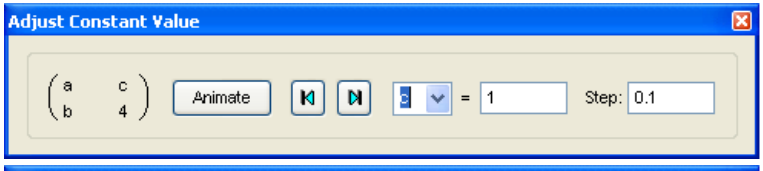
PRE-SETS: 2 x 2 Matrices listed here by columns

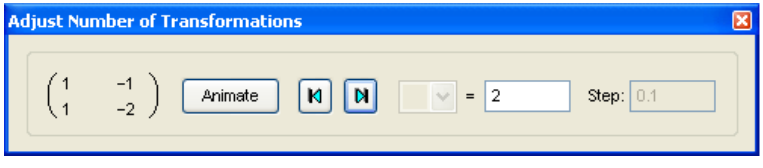
Enlargement	(k	0	0	k)	Δ = k^2
Reflection, x = 0	(–1	0	0	1)	Δ = –1
Reflection, y = 0	(1	0	0	–1)	Δ = –1
Reflection, y = x	(0	1	1	0)	Δ = –1
Reflection, y = –x	(0	–1	–1	0)	Δ = –1
Reflection, θ	(cos2θ	sin2θ	sin2θ	–cos2θ)	Δ = –1
Rotation, θ	(cosθ	sinθ	–sinθ	cosθ)	Δ = 1
Rotation, –90°	(0	–1	1	0)	Δ = 1
Rotation, 30°	(√3/2	1/2	–1/2	√3/2)	Δ = 1
Rotation, 90°	(0	1	–1	0)	Δ = 1
Rotation, 180°	(–1	0	0	–1)	Δ = 1
Stretch	(a	0	0	b)	Δ = ab
Shear // x-axis	(1	0	k	1)	Δ = 1
Shear // y-axis	(1	k	0	1)	Δ = 1
Singular	(a	b	ka	kb)	Δ = 0

A = (a, b, c, d) Determinant: Δ = ad – bc.
Eigenvalues: λ = two roots of $k^2 – (a+d)k + Δ = 0$
Eigenvectors, the two equations: y = (b/(λ – d))x [λ ≠ d]







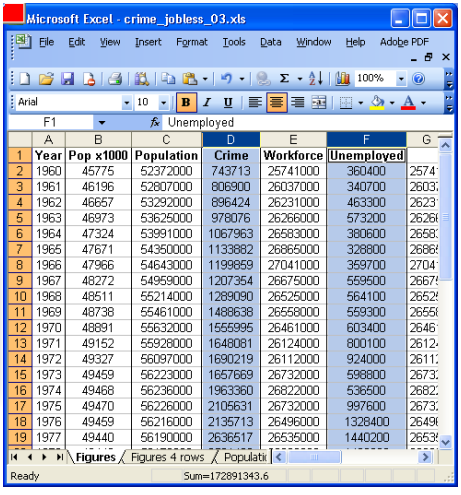


The Animate option

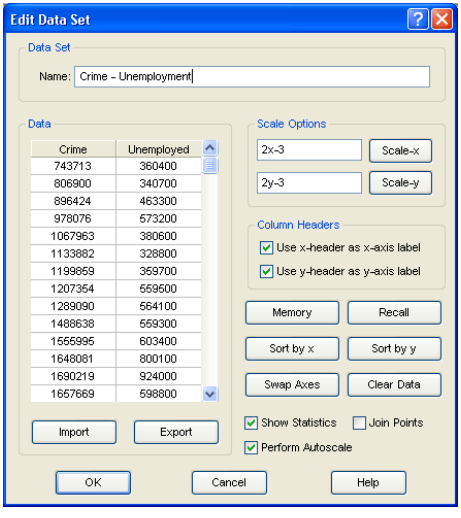
If the elements include one or more **constants**, use this option to animate their values and display the matrix.

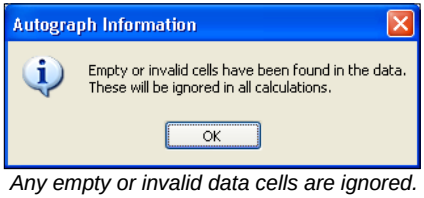
If the elements do NOT include any constants, use this option to repeat the transformation, i.e. A^n for n = 2, 3, 4 or backwards to 1, 0, –1, etc.

2D: XY (Bivariate) Data



	A	B	C	D	E	F	G
1	Year	Pop x1000	Population	Crime	Workforce	Unemployed	
2	1960	45775	52372000	743713	25741000	360400	2574
3	1961	46196	52807000	806900	26037000	340700	2603
4	1962	46657	53292000	896424	26231000	463300	2623
5	1963	46973	53625000	978076	26266000	573200	2626
6	1964	47324	53991000	1067963	26583000	380600	2658
7	1965	47671	54350000	1133882	26865000	328800	2686
8	1966	47966	54643000	1199859	27041000	359700	2704
9	1967	48272	54959000	1207354	26675000	559500	2667
10	1968	48511	55214000	1289090	26525000	564100	2652
11	1969	48738	55461000	1488638	26558000	559300	2655
12	1970	48891	55632000	1555995	26461000	603400	2646
13	1971	49152	55928000	1648081	26124000	800100	2612
14	1972	49327	56097000	1690219	26112000	924000	2611
15	1973	49459	56223000	1657669	26732000	598800	2673
16	1974	49468	56236000	1963360	26822000	536500	2682
17	1975	49470	56226000	2105631	26732000	997600	2673
18	1976	49459	56216000	2135713	26496000	1328400	2649
19	1977	49440	56190000	2636517	26535000	1440200	2653





Any empty or invalid data cells are ignored.

In Autograph, an XY data set is a single object made up from a number of points. You can have several data sets on the same page, and each has a different colour.

"ENTER XY Data Set"

This dialogue box offers three basic ways to create a data set:

PASTING IN FROM A SPREADSHEET

First select two columns of data (e.g. columns 'D' and 'F' in the illustration). If the two columns are not adjacent use CTRL as you select the second column. Unfortunately, any intermediate columns are also put on the clipboard. To get around this, paste the selection into a new spreadsheet page, select and copy again.

Back in Autograph, in the Edit Data Set dialogue box, click on the top left empty cell and paste in the data (CTRL-V, or use the right-click "paste" option). The column headers are entered automatically. To edit them, click on a cell in one column and use the right-click option.

○ MANUAL ENTRY

Enter the points manually: click on the first cell, enter the first item, then press ENTER then the next, then press ENTER, etc.

IMPORT (CSV):

Use this option to import data that has been saved as 2-column CSV file (comma-separated variable). Likewise to Export.

♦ **Scale Options:** Use this to apply a formula to either column. 'x' refers to the left column, 'y' to the right.

♦ **Column Headers:** To use either or both headers as the axes lables, tick the options.

♦ **Memory:** Use this to store a data set, e.g. if you want to scale it and then return to the original values (using "Recall").

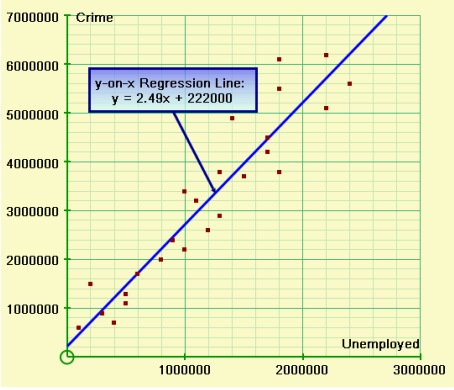
♦ **Sorting:** use these options to sort either column.

♦ **Swap:** use this to swap the axes if the wrong one is the independent variable (as here).

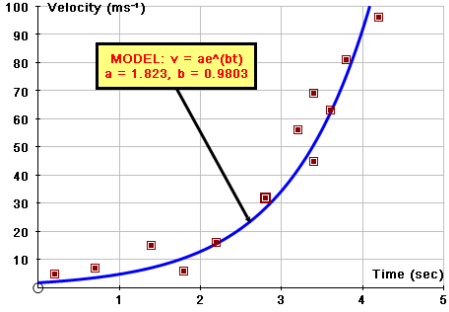
♦ **Show Statistics:** tick this to add a statistics results box (see opposite). More detail is given at Advanced level.

♦ **Join Points:** Use this option to join up points. Only suitable for 'sequential' data.

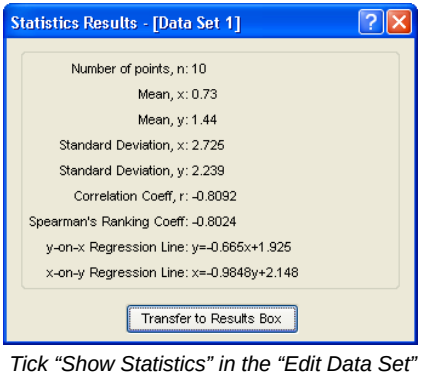
♦ **Perform Autoscale:** this ensures that all the data appears on the graph axes when plotted (ticked by default).



A linear data set derived from 2 columns in Excel, with Regression line and text box.



Using the "Best Fit to Data" option: in this case best values of 'a' and 'b' were found.



Tick "Show Statistics" in the "Edit Data Set" dialogue box, to obtain this information.

THE DATA SET OBJECT (Scatter Diagram)

On exiting "Edit Data Set", the points are plotted automatically as a data set object (Scatter Diagram) with a new colour. With "Perform Autoscale" ticked in the dialogue box, the data will all appear on the page. If not, you may need "Default Scales" for the data to appear.

♦ **To select or edit the data set:** Click on *any one point* to select all the points as a single object. *Double-click* on any one point to return to the "Edit Data Set" dialogue box. At this point, data can be added, deleted, scaled, etc.

♦ **To select individual points:** use *CTRL* and click, or use the rectangular marquee selection (when in 'select' mode).

With a DATA SET selected a number of options are available**:**
♦ **Centroid**: shows $(\bar{x}, \bar{y})$ as a calculated point (small square).
♦ **Line of best fit** [Standard Level]
♦ **y-on-x or x-on-y regression line** [Advanced Level]

♦ **Transformation -** by the standard set of transformations: **Reflection**, **shear** and **stretch** in x- and y-axes; **Matrix** [Advanced Level]. In each case, a new, grey, dataset is shown. Also: **Reflection** (with a selected line), **Translation** (with a selected vector); Rotation, **Enlargement** (with a selected point).

With a DATA SET and ANY STRAIGHT LINE selected:
♦ **x-on-y residuals; y-on-x residuals** (Advanced Level).
Options: lines or squares; ensure square aspect; $\sum r^2$ given.

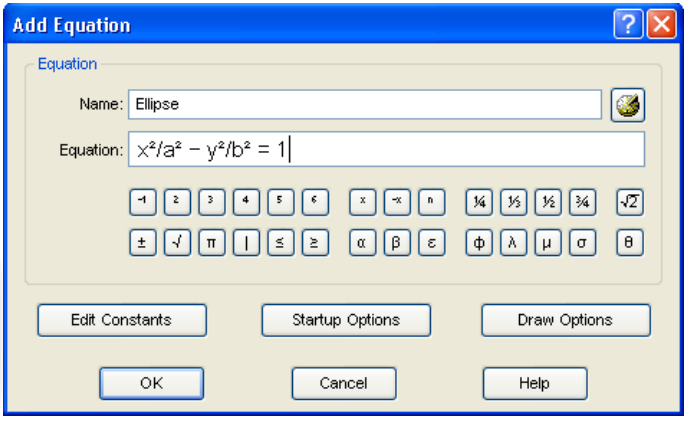
With a DATA SET and ANY y = f(x) graph with constants:
♦ **Best fit to Data**
This will find (by least squares) the best value of the constants to fit the data, e.g. y = asin(bx + c) + d. Select the graph and use the option "**Update Results Box**" to obtain a list of the constants.
If a polynomial fit is required (up to x^6), select the data set and use the option "**Best Fit ..**"

Other ways to CREATE/EXTEND A DATA SET

Select a number of points, and use the option "**Convert to Data Set**". You can also select some free points AND a data set, and use the option "**Add to Data Set**". "Revert to points" reverses this.

Select a y = f(x) graph, and use the right-click option "**Create Data Set**". This is the best way to perform a geometric transformation on a plotted graph.

2D: Entering Equations



Click this button, or press ENTER any time, or use the right-click "Enter Equation" to bring up this dialogue box.

Equations are entered as text, so can be pasted in from elsewhere.

Mathematical symbols come from the *Arial for Autograph Uni* font. Either use the buttons provided, or use the ALT key shortcuts below, which will work in all of Autograph's dialogue boxes.

Alternatively, you can use the on-screen keyboard.

DIRECT ENTRY OF MATHEMATICAL SYMBOLS

"xx" enters x^2, "xxx" enters x^3, etc., up to x^6. The following keyboard shortcuts can be used to enter special characters at ***any input box in Autograph***: hold down ALT and tap the other:

ALT	1	2	3	4	5	6	x	y	n
Indices	x^{-1}	x^2	x^3	x^4	x^5	x^6	e^x	e^y	x^n

ALT	A	B	E	F	L	M	P	S	T
Greek	α	β	ε	ϕ	λ	μ	π	σ	θ

ALT	R	+	I	H	<	>	'PI' also enters π
Maths	Ä	±	|	½	≤	≥	<= and >= enter ≤ and ≥

Notes: For all other indices, use ^ (SHIFT-6), e.g. y = e^sinx
Greek letters can be 'constants' in equations, apart from θ (which is reserved as a parameter).

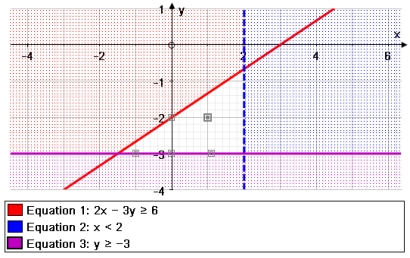
The following mathematical operators are used in Autograph:

sin	cos	tan	cosec	sec	cot	
$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$	cosec^{-1}	$\sec^{-1}$	$\cot^{-1}$	[or arcsin, etc]
sinh	cosh	tanh	cosech	sech	coth	
$\sinh^{-1}$	$\cosh^{-1}$	$\tanh^{-1}$	cosech^{-1}	sech^{-1}	$\coth^{-1}$	[or arcsinh, etc]
log	ln	e	Brackets are optional, e.g. $y = 2\sin^2 2x$ or $y = 2\sin^2(2x)$			

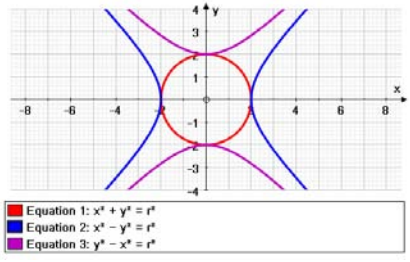
EDIT CONSTANTS

Click here to set the initial values of any constants in the equation, if necessary. They are all pre-set to start =1. Once the graph is drawn, the **Constant Controller** can vary the constants dynamically.

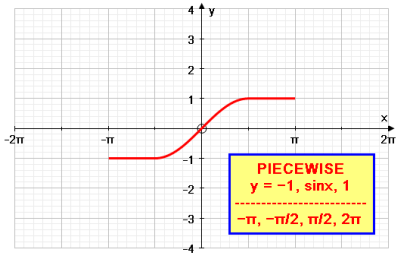
2D: Cartesian Equation Types



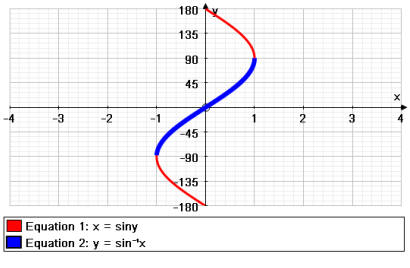
Inequalities



2nd degree equations (conics)



Piecewise function



Inverse trig function

STARTUP OPTIONS and DRAW OPTIONS

There are various equation types that Autograph understands, and each have their own startup options (see below). With the **Draw Options** you can vary colour, thickness and line style (eg dashed).

---

y = f(x), or equations that simplify to y = f(x)

♦ **Straight lines:** $y = mx + c$ $ax + by = c$

♦ **y = f(x):** $y = x^2(x - a)$ $y = a\sin(bx)$

♦ **Inequalities:** $2x + 3y \leq -2$ $y < \sin x$

'≤' or '≥' draws with a solid line; '<' or '>' draws with a dotted line
The default is to shade the 'reject' region, but this can be set to the 'accept' region in "View" => "Preferences".

♦ **2nd degree:** $y^2 = x(3 - 2x)$ $x^2 = y^2$

$(x - a)^2 + (y - b)^2 = r^2$ $x^2/a^2 - y^2/b^2 = 1$

$ax^2 + by^2 - 2gx - 2fy + hxy + 1 = 0$

These are all arranged internally as y = ±Ä(...) and plot as two separate functions, each of which can be selected separately.

STARTUP OPTIONS: For all of the above, the default is to plot with automatic step from x-min to x-max. You can switch this to 'manual' and control the domain, e.g. to plot just a segment of a graph.

The General Implicit

♦ **f(x, y)** $\sin x = \cos y$

These are generated by a special process, and the resulting plots are not 'functions', so cannot be selected, or have points placed on them.

STARTUP OPTIONS: you can set the intensity of the scanning from 1 to 200 in either, or both, directions. The default is 100.

Piecewise Expressions

♦ **y = ..., ..., ...** $y = \sin x, \cos x, 2 - x$

Enter several functions, each separated by a comma.
STARTUP OPTIONS: You set the start 'x' value for each function, and the final end value. The resulting function plots as if it was a single graph, and can be treated as such (e.g. gradient, intersections, etc).

---

x = f(y), or equations that simplify to x = f(y)

eg $x = 2$ $x = \ln(y)$

$x^2 = y^2$ $x^3 = \sin y$

'y' becomes the independent variable.

2D: Objects Related to Graphs

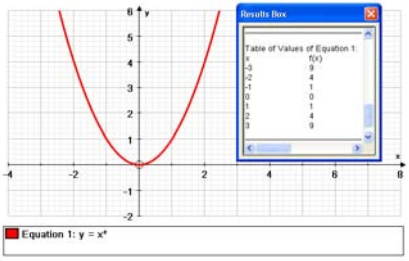
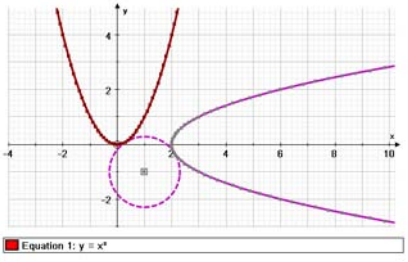
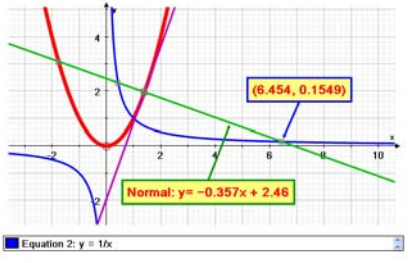


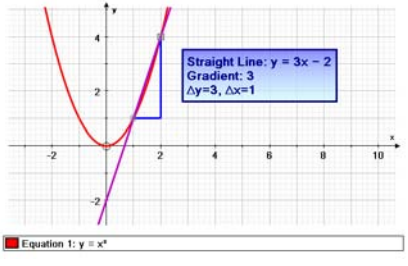
Table of values (in the results box)



Function as a data set, then rotated



Coordinate geometry



Two points on $y = x^2$: gradient triangle

WITH A GRAPH SELECTED you can:

♦ **Update Results Box:** This places the equation and the values of any constants in the Results Box.

♦ **Table of Values:** enter range and step, and the table is created as 2 TAB-separated columns of text, which can be copied out to Excel.

♦ **Create a Data Set:** enter range and step, and this places a data set of joined up points on the graph. If selected, this data can be transformed just like a shape: translation, reflection, shear, rotation, matrix, etc.

♦ **Draw the Evolute:** enter range and step to draw a family of tangents or of normals to the selected curve.

♦ **Asymptote:** draws a tangent at a very large value of 'x'.

♦ **Solve f(x) = 0:** this places a (purple) marker point on all the solutions in the plotted range, and puts a list in the Results Box.

♦ **Solve f'(x) = 0:** this places a (purple) marker point on all the turning points in the plotted range, and puts a list of the answers in the Results Box. You can add a point to any such marker point if required.

♦ **Enter point on curve:** enter 'x' coordinate [or 'y' for x = f(y) graph].

WITH ONE POINT ON THE GRAPH

All the coordinate geometry options for a single point (e.g. vertical, line, vector, etc.) together with:

♦ **Tangent**	Equation given as y = mx + c
♦ **Normal**	Equation given as y = mx + c
♦ **Move to next f(x) = 0**	Moves to the right
♦ **Move to next f'(x) = 0**	Moves to the right

♦ **Attach to object:** available if you select a point NOT on a graph. The point takes its value of 'x' to the graph and becomes attached.

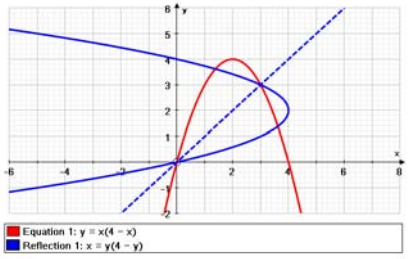
WITH TWO POINTS ON THE GRAPH

♦ **Gradient** (right angled triangle) *Δy, Δx; equation of line; Δy/Δx*

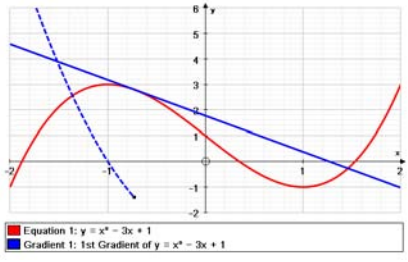
WITH TWO GRAPHS SELECTED

♦ **Solve f(x) = g(x):** "Calculated points" (small coloured square) are placed at all intersections. A list of the coordinates is posted in the Results Box. You can add a point to any of these if required.

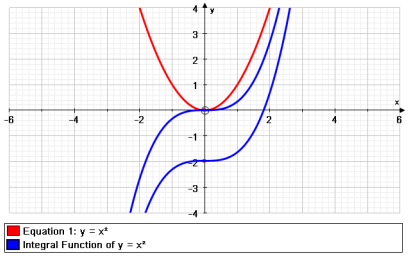
CTRL with the Point Mode
While holding CTRL, wait for the mouse cursor to change to a circle over an intersection or max/min, then click to place a point there.



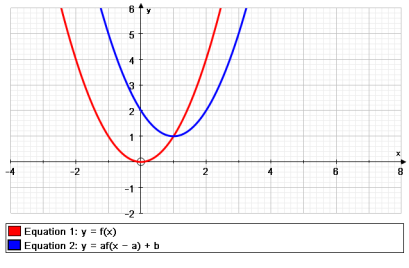
Reflection in y = x



1st Derivative created by the tangent slope



Integral function



Use of function definitions

THREE 'CREATE' OPTIONS
for the currently selected y = f(x) Graph
as listed in "Equation" => "Create"

1. REFLECTION in y = x

This will draw y = x as a dotted line together with the reflection of the current graph. Its new equation is given in the status bar, obtained by swapping 'x' and 'y' in the current equation. The resulting graph may or may not be an 'inverse' function.

---

2. GRADIENT (Slope) Function [Advanced Level only]

This will create the first derivative of the currently selected equation, plotted as a dotted line.

If Slow plot is ON, the gradient plots slowly, showing a moving tangent on the parent graph. Plotting pauses at all turning points on the way. Press the **spacebar** to continue or pause at will.

---

3. INTEGRAL FUNCTION [Advanced Level only]

By default, this waits for mouse clicks to set the initial value (x_o, y_o) for each solution. e.g. $y = x^2$ will give solutions of the form $y = \frac{1}{3}x^3 + c$.

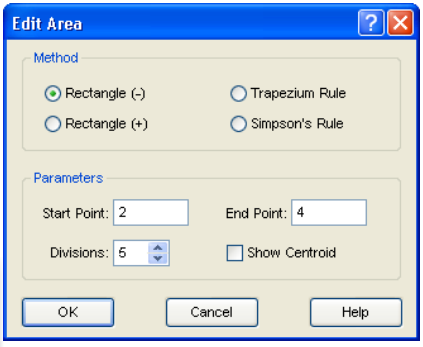
This is effectively solving the 1st order differential equation $dy/dx = x^2$, and the start-up options for an integral function are the same as those for a 1st order Differential Equation (see page 62). With Slow Plot on, the solutions are generated slowly.

DEFINING FUNCTIONS f(x) and g(x)

Use this option to enter an expression in 'x' for f(x) and or g(x). Either can also include a reference to the other, for example:

Definition:	*Equation entry:*
$f(x) = x^2$	$y = f(x)$
	$y = f(x + a)$
$g(x) = (f(x + h) – f(x – h))/(2h)$	$y = g(x)$

2D: Numerical Methods 1 - Areas



AREA between a graph and the axis.

This right-click option is available on the selection of

♦ y = f(x) graph, OR two points ON a y = f(x) graph,

OR

♦ x = f(y) graph, OR two points ON a x = f(y) graph.

The dialogue box offers four options, using start and end values of 'x' (or 'y') that are entered, or taken from the two selected points.

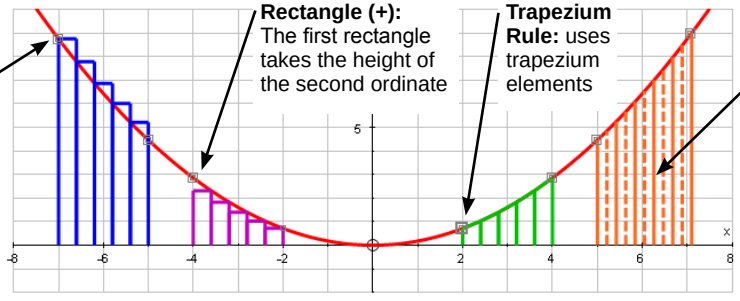
"**Show Centroid**" will mark $(\bar{x}, \bar{y})$ as a "calculated" point.

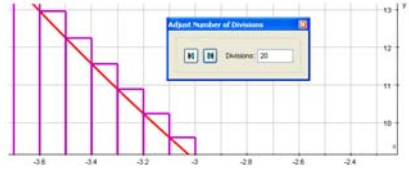
Rectangle (–): The first rectangle takes the height of the first ordinate

Rectangle (+): The first rectangle takes the height of the second ordinate

Trapezium Rule: uses trapezium elements

Simpson's Rule: uses quadratic elements



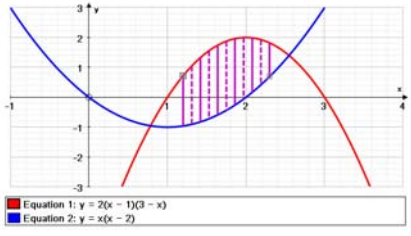


An area with the number of divisions increased using the Animation tool, and the graph zoomed in.

An area defined by two POINTS can be varied by moving either point.

To select an area, click on one of its vertical lines
To re-edit an area, double-click on one of its vertical lines

With an area selected, the ANIMATE button allows the number of divisions to be animated.



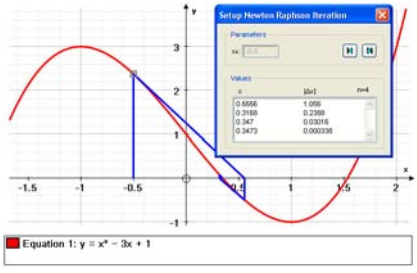
An area between two graphs, with the number of divisions increased, and the graph zoomed in.

AREA between two graphs

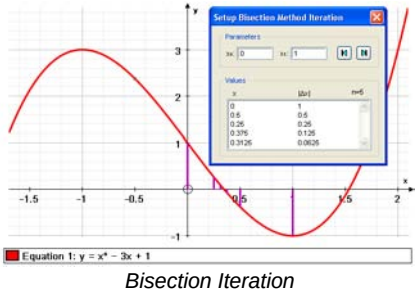
EITHER: select two y = f(x), or x = f(y) graphs (order is significant), and use the right-click option to enter the start and finish values.

OR: select a point ON the first graph, and then a point on the second graph (the order is significant), then use the right-click option.

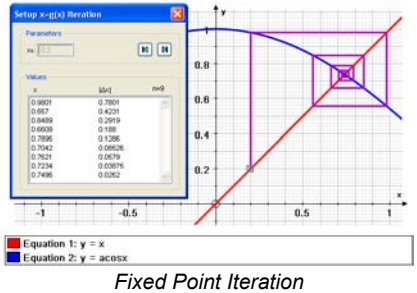
The two points must be associated with each of the graphs, so to use the intersection points, put the points on to the LEFT of each intersection. Then for each, select one point and the OTHER graph, and use the option "move to next intersection".



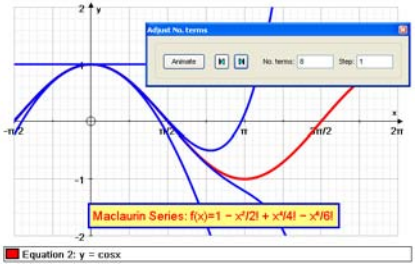
Newton-Raphson Iteration



Bisection Iteration



Fixed Point Iteration



Maclaurin Series

♦ NEWTON-RAPHSON ITERATION

EITHER select a y = f(x) graph:
The right-click option opens a dialogue box which controls the iterations. The initial value x_o is entered. Use the arrow buttons or use the UP/DOWN arrow keys to add or subtract iterations.

OR select a point ON a y = f(x) graph
Create iterations as above. You can then move the point about to vary the iteration.

To select an iteration: click on any of its elements.
To re-edit an iteration: double-click on any of its elements.

♦ BISECTION ITERATION

EITHER select a y = f(x) graph,
The right-click dialogue box controls the iterations. The initial values x_o and x_1 are entered. Use the arrow buttons or use the UP/DOWN arrow keys to add or subtract iterations.

OR select TWO points ON a y = f(x) graph.

♦ FIXED POINT ITERATION: x = g(x)

EITHER select the graph y = x AND a y = f(x) graph

The right-click dialogue box controls the iterations. The initial value x_o is entered. Use the arrow buttons or use the UP/DOWN arrow keys to add or subtract iterations.

OR select a point ON y = x, and a y = f(x) graph.

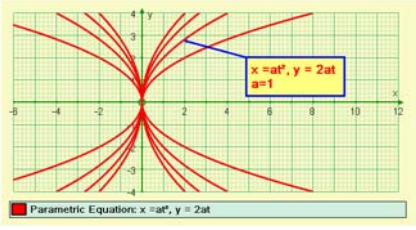
The f(x) graph can contain a constant, so the whole iteration can be altered by varying the value of the constant.

♦ MACLAURIN SERIES

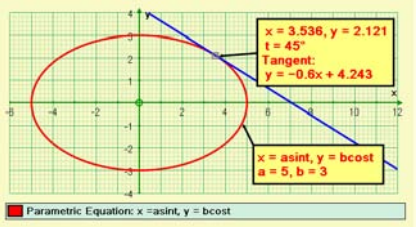
Select a y = f(x) function. Use the right-click option to specify the number of terms (max 10) to display, with or without the intermediate terms. For larger 'n' the numerical method used can get unreliable.

The status bar gives the terms. Click on any one of the new curves to select it. The Animation controller can be used to vary the number of terms dynamically.

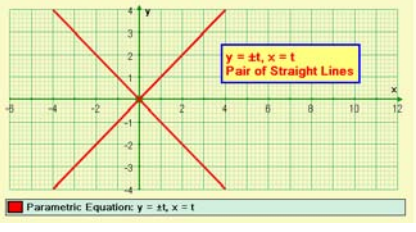
2D: Parametric Graphing



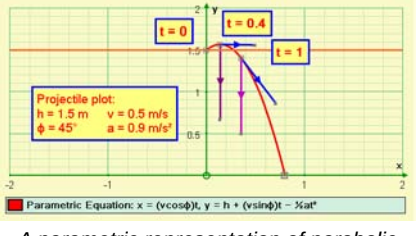
A family of parametric curves. Use the "Options" button on the constant controller.



A trigonometric plot using parametric coordinates - in this case set to degrees.



A pair or straight lines plotted parametrically. The ± creates two plots.



A parametric representation of parabolic motion under gravity. Use ALT-F to enter φ.

PARAMETRIC GRAPHS:
Use a comma to separate x(t) and y(t), either way round.
e.g.: $x = ut, y = h + vt - \frac{1}{2}gt^2$ A parabola
$x = a\sin\theta, y = b\cos\theta$ A circle or ellipse
You can use 't' or 'θ' [ALT-T] as the parameter.

Startup Options:
Auto: this will do the best to ensure that the graph is plotted successfully. Typically this starts at t = 0, draws until the plot goes off-screen, then starts again at t = 0 the other way. With trig functions, the defaults become t = 0 to 360°, or 0 to 2π.
Manual: set up the start, finish and step.

NOTE: The auto setting will assume the trig settings if a trig function occurs in the equation. Sometimes this is inappropriate, e.g. $x = (v\cos\phi)t, y = (v\sin\phi)t - \frac{1}{2}gt^2$ when manual settings for 't' are essential. Also, these trig expressions use a 'constant' ϕ, and θ must be reserved for the parameter variable (shared with 't').

Slow plotting: you can pause (spacebar) and read off values of 't'. The Status bar shows 't', 'x' and 'y' values, to the current sig fig.

With a PARAMETRIC GRAPH selected:

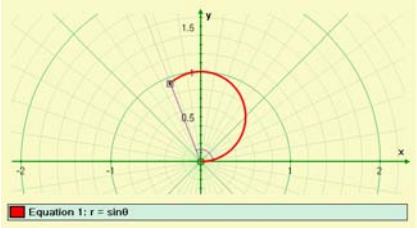
♦ **Table of Values**	Set 't' range and step. 't', 'x', 'y' are listed in the Results Box.
♦ **Create Data set**	Set 't' range and step. A joined up XY data set is created.
♦ **Asymptote**	Draws tangent at high value of 't'
♦ **Evolute**	Draws family of tangents or normals

Gradient (Slope) Function
This will generate, with a dotted line, the first derivative, dy/dx, for the current parametric curve (as selected in the Equation Key).

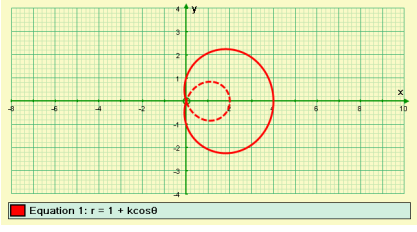
A **point added** to a parametric curve will be added to the nearest 'nice' value of 't'. The LEFT/RIGHT arrow keys can then be used to move a point along the curve with the current step. Double-click on such a point to edit the current value of 't' and the value of the current step.

With a POINT attached to a Parametric curve:
(in addition to the usual one point options)

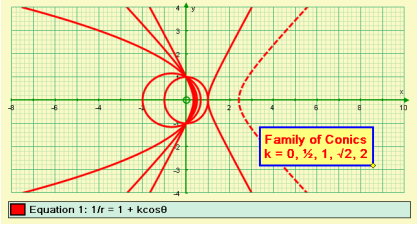
♦ **Tangent**	Equation given as y = mx + c
♦ **Normal**	Equation given as y = mx + c
♦ **Velocity vector**	Coordinates given
♦ **Acceleration vector**	Coordinates given



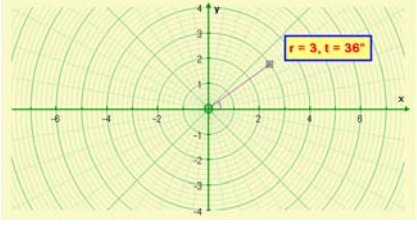
Plotting a polar graph slowly gives a chance to show the value of 'r' and 'θ'



When values of 'r' go negative the plotting is dotted



A family of conics using a comma separated list of values for 'k'



A selected point on a polar graph page shows its values of 'r' and 'θ' dynamically.

POLAR GRAPHS: Any of these forms can be used:

e.g.:	r = a	A circle
	r = sin2θ	A 'flower'
	1/r = 1 – kcosθ	A conic
	r^2 = t	Two 'branches'
	θ = nπ/8	A family of radial lines

You can use 't' or 'θ' [ALT T] as the parameter.

Use Polar Axes and Equal Aspect for the best effect.

Use Edit Axes "Options" tab to add grid sub-divisions.

Startup Options:
Auto: this will plot θ = 0 to 360° (degrees), or 0 to 2π (radians).
Manual: use this to control the start, finish and step.
Plotting indicates negative values of 'r' with a dotted line.

Slow plotting: you can pause and read off values of 'θ'. 'r' and 'θ' are indicated visually. The status bar shows 't', 'x' and 'y' values.

With a POLAR GRAPH selected (as with a Parametric):
♦ Table of Values, ♦ Create Data Set, ♦ Asymptote, ♦ Evolute.

Gradient (Slope) Function
This will generate, with a dotted line, the first derivative, dy/dx, for the current polar curve (as selected in the Equation Key)

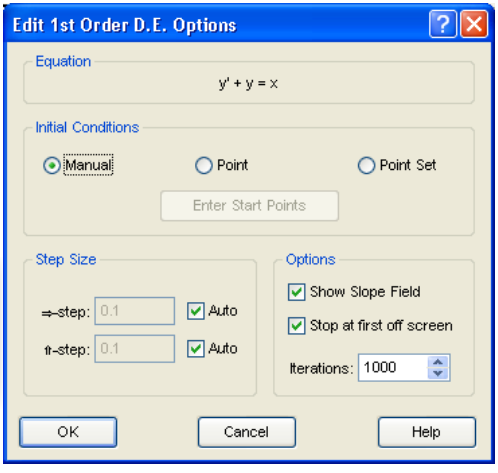
Default snap settings for 't' (or 'θ'): π/32 (radians), 4° (degrees)
These are used for dragging polar points about:

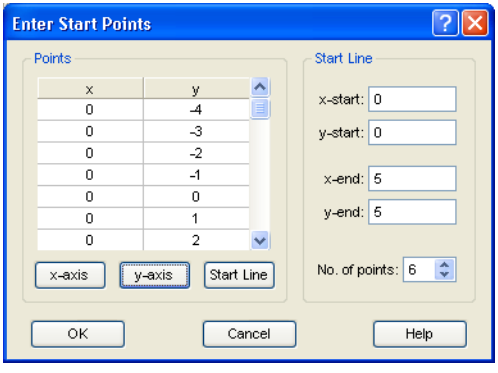
Points attached to a polar graph
A point added to a polar curve will be added to the nearest 'nice' value of 't' set by the default 'snap setting'. The LEFT/RIGHT arrow keys can then be used to move a point along the curve with the current step. Double-click on such a point to edit the current value of 't' and the value of the current step.

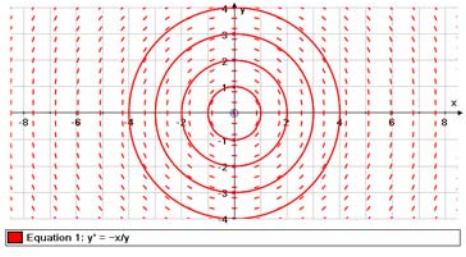
With a POINT attached to a Polar curve (as with a Parametric):
♦ **Tangent,** ♦ **Normal** Equations given as y = mx + c

Free points on polar axes
A free point added to a polar axes, and selected: the point indicates 'r' by joining to the origin, and 'θ' by showing the angle. The point can be dragged about using the mouse, or moved using the ARROW keys:
LEFT/RIGHT ... vary value of 'θ', by the current step
UP/DOWN ... vary value of 'r', by the current step
(+ Ctrl: , + Shift to vary the step /10 and /100 as usual).

2D: Differential Equations







The Runge-Kutta numerical method is adequate for most 1st order situations. Here, the solutions are circular.

1st ORDER DIFFERENTIAL EQUATIONS

Equations can be entered using y' *or dy/dx, eg:*

Variables y-x:	y' = y/x	y' + ky = x	dy/dx = x
Variables x-t, or v-t:	$\dot{x} = 2t^2$	$\dot{v} = \sin t$	

Use "Edit Axes" to set the variables, or ...

Use "Re-label x-t" to set the axes quickly to x-t

With independent variable 't': x' and v' become $\dot{x}$ and $\dot{v}$.

STARTUP OPTIONS: If you are happy with the default settings, there is no need to visit this dialogue box.

♦ Initial Conditions

Manual - solutions are started for each mouse click. These are plotted slowly if "Slow Plot" mode is on.

Point - for this you need a selected single point before you enter the start-up options. A solution is drawn from this point, which can be dragged about.

Point Set - use the "**Enter Start Points**" to set up a number of start points in a straight line, either along one of the axes, or between two entered points.

♦ Step Size

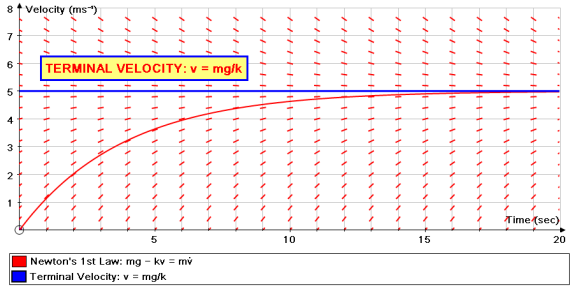
The "Runge-Kutta" method is employed, and it works fine in most cases, switching from x-stepping to y-stepping when the slope gets steep. Here you can control the x- or y- step if required. With "Auto" ticked, solutions are generated in positive and negative directions from the start point.

♦ Options:

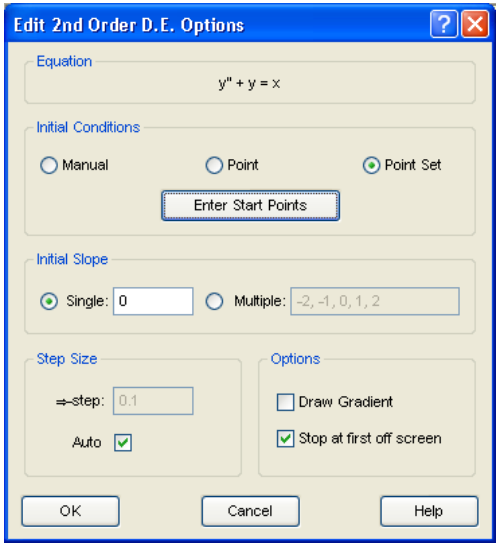
Show Slope Field - 21 instances of the slope in each of the 'x' and 'y' directions are indicated by small tangents.

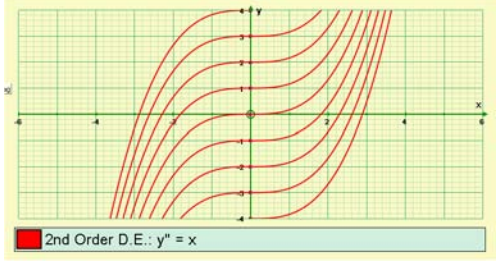
Stop at first off-screen - solutions can plot forever off-screen.

Iterations - a safeguard to prevent solutions going on forever.

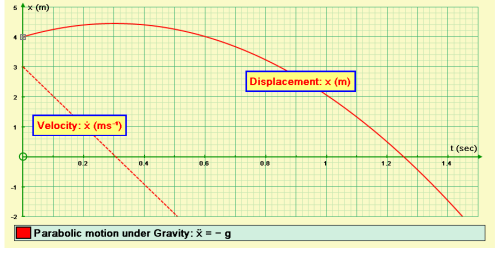


Here, the variables have been changed to 'v' and 't'. Entering v' will change automatically to $\dot{v}$. *The slope field is optional.*





A nice illustration of the integration process.
y" = x => y' = ½x² + c => y = ⅙x³ +cx + d



A representation of motion under gravity.
All the computer has been told is: ẍ = −g
and the intitial conditions: x(0) = 4, ẋ(0) = 3

2nd ORDER DIFFERENTIAL EQUATIONS

Equations can be entered using y" or d²y/dx² , e.g.:

Variables y-x: y" = x y" + y' = 3
Variables x-t: ẍ + 2λẋ + x = sint

Use "Edit Axes" to set the variables, or ...
Use "Re-label x-t" to set the axes quickly to x-t
When the independent variable is 't': x', x" become ẋ and ẍ.

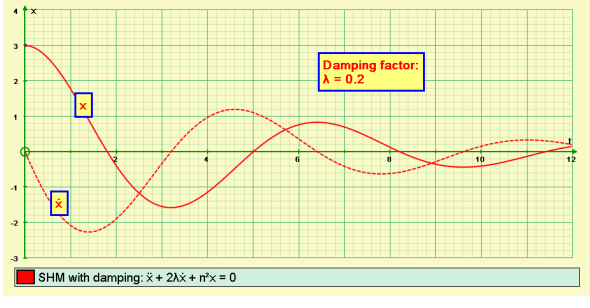
STARTUP OPTIONS: If you are happy with the default settings, there is no need to visit this dialogue box. *A second order equation requires two initial conditions - Autograph uses an initial point and initial slope.*

♦ Initial Point
Manual - solutions are started for each mouse click.
Point - for this you need a selected single point before you enter the start-up options. A solution is drawn from this point, which can be dragged about.
Point Set - use the "Enter Start Points" to set up a number of start points in a straight line, either along one of the axes, or between two entered points.

♦ Initial Slope
Here, set the initial value, or a comma separated list.

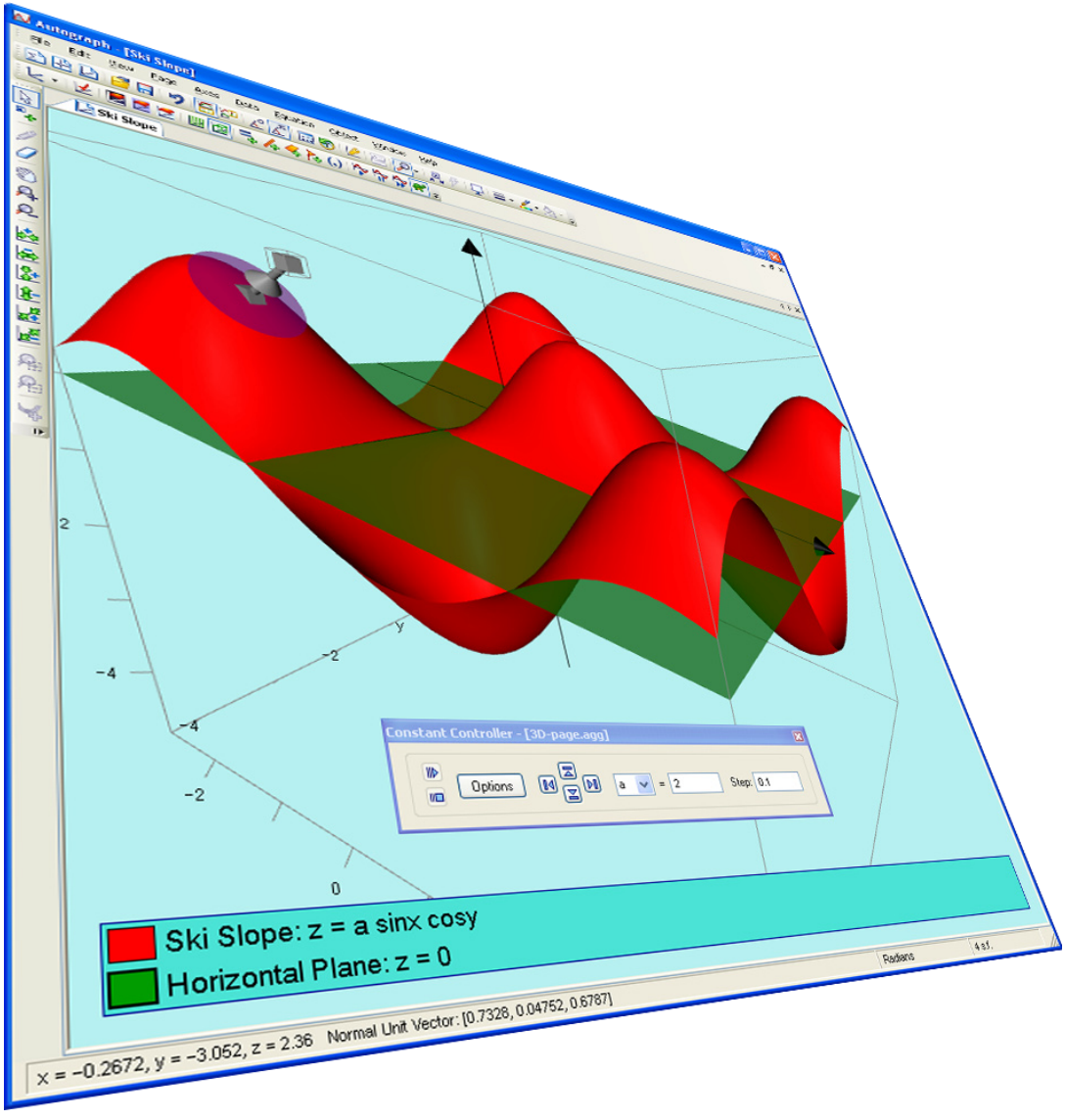
♦ Step Size
A modified "Runge-Kutta" method is employed, and it works fine in most cases. Only the independent variable, 'x' or 't', is stepped, and you can control the step if required.

♦ Options:
Draw gradient - this plots the 1st derivative, dy/dx or dx/dt, along with the main solution.
Stop at first off-screen - solutions can plot forever off-screen.

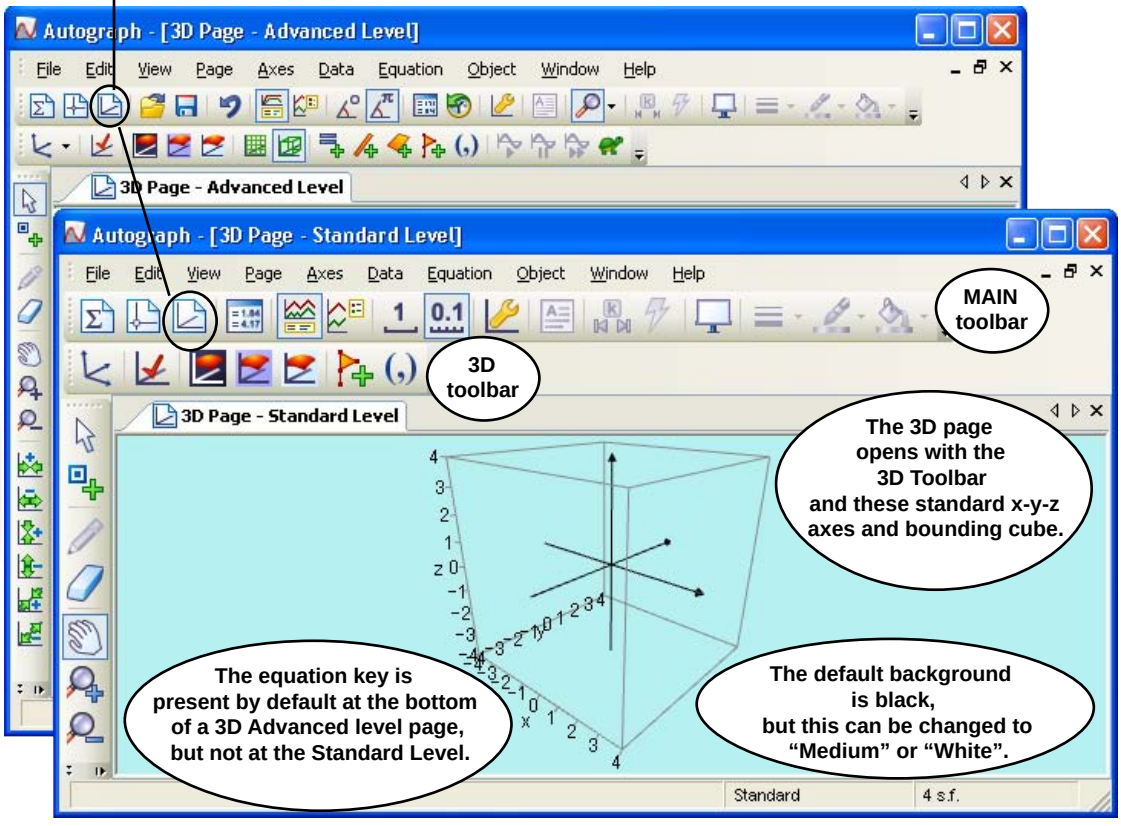


Damped SHM. The values of 'λ' (damping factor) and 'n' (natural frequency) can be varied. Note the implicit form for the differential equation.

3D PAGE: XYZ Graphing

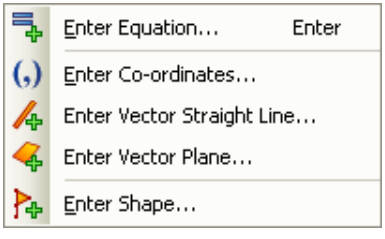


3D: Standard and Advanced



3D

THE 3D PAGE:
The TOP LEVEL
RIGHT CLICK MENU



- **Enter Equation** **[Advanced Level only]**
  [Pressing ENTER also opens this]
  The type of equation is detected automatically. *Constants can be included; Start-up options will depend on the type of equation.*

- **Enter Co-ordinates**
  You can use numbers, expressions or constants.

- **Enter Vector Line** **[Advanced Level only]**
  A line in the form: $[x,y,z] = [a,b,c] + \lambda[g,f,h]$

- **Enter Vector Plane** **[Advanced Level only]**
  A plane in the form: $[x,y,z] = [a,b,c] + \lambda[f,g,h] + \mu[l,m,n]$

- **Enter Shape**
  A choice of 'preset' shapes, or enter a set of coordinates.

3D: Graphing Toolbar

THE 3D MODES TOOLBAR

The Modes toolbar has been modified to suit the needs of the 3D environment.

Select mode: there are no mouse indications on a 3D page (to improve speed). When not hovering over an object, the select mode behaves like the **Drag tool**.

Point tool: A point is placed in 3D halfway through the bounding cube, along the eye-line. Double-click on a point to edit its coordinates.

[It is not practicable to draw in 3D!]

Rubout tool

Drag tool: This rotates the whole scene about.

+ CTRL: DOWN/UP
This zooms the camera IN/OUT

+ SHIFT:
This moves the 'camera' about.

Zooms: zooms all the axes, centre in the middle of the cube along the eye-line

This zooms only the x-axis

This zooms only the y-axis

This zooms only the z-axis

THE 3D GRAPHING TOOLBAR

Restores the x-y-z **orientation** to the default/
This is particularly useful after using the Drag tool with CRTL to restore the original orientation.

Hides the z-axis for **y-x plotting in 3D**, and re-orientates with 'y' axis vertical. This is particularly useful for Volumes of Revolution.

Sets or restores **default ranges.**
Useful if in a muddle after zooming.
Will put 'x' and 'y' π scales in for trig functions.

Dark **background** (default) - good for surfaces.

Medium **background** - good for Whiteboards.

White **backgrounds.**
each has amended axes' colours to match.

Show/hide **3D grid.**

Show/hide **3D box.**

Add **Equation (*)**

Add **vector equation of a line (*)**

Add **vector equation of a plane (*)**
(*) Advanced Level only.

Add **3D shape** (including defaults).

Add **Coordinates for a point.**

Revisit the start-up options and **replot** a graph.

- used to control 3D **object animation.**

STANDARD and ADVANCED 3D

The Standard level in 3D does not offer equations, or the vector equations of a line, or a plane, but otherwise the two levels operate in the same way.

There is a 'bounding cube' inside which all the action takes place. It is not possible to scribble, or put on a text box in 3D as both would have to be fully implemented 3D objects.

USEFUL TOOLS in the MAIN Toolbar

Key below/right, works as in 2D.

Angle settings.

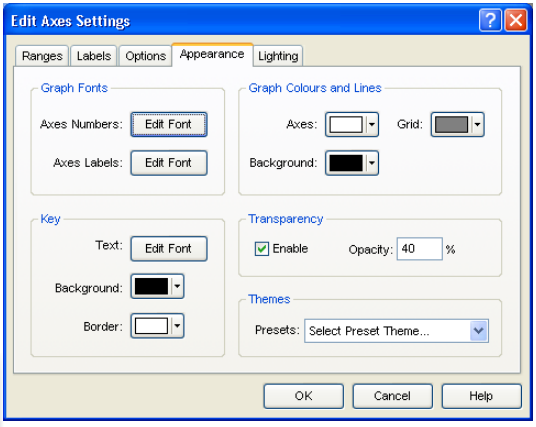
Constant controller and Animation.

Thickness applies to lines only.

Standard Level only: Snap=1 is the default.

3D: How Does It Work?

EDIT AXES in 3D



Ranges, Labels, Options:

These are the same as 2D, except the 'Options' defaults are:
Axes: always outside, with arrows and a bounding box.

APPEARANCE:

Background: it is usually better to use the three toolbar buttons, as the axes colours are adjusted to fit automatically.

Transparency: the 3D equation Entry "Draw options" allows colour and transparency of surfaces to be set, but generally it is better to set transparency for all surfaces (here) rather than just one.

Lighting: Here you can set "Ambient Light", "Reflectance" and "Sharpness", as well as add more lights. What you can do will depend on the capabilities of your graphics card.

DRAG and ZOOM in 3D

This very versatile 'drag' feature is duplicated in select mode when not over an object. You can drag the scene round with the left mouse button pressed.

+ CTRL: you can zoom the 'camera' in and out of the scene by moving the mouse UP and DOWN.
+ SHIFT: you can move the 'camera' about.

ADDING POINTS IN 3D

Points in the 3D environment are represented by small cubes, and selected points have a small extra 'layer'. There is no distinction given to the 'current' point.

The point mode is used to add points just as in 2D. When the mouse is NOT over an existing object, the points are '**free**', and follow the rule that they are placed halfway through the bounding cube. The coordinates can of course be edited (double-click).

A point added while the mouse is over an object (eg a plane) is '**attached**' to the object. Double-click on an attached point to **edit** the coordinates (usually only the 'x' and 'y' values).

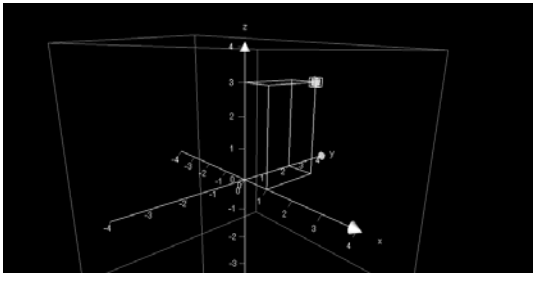
(,) Use "Enter coordinates" to enter exact coordinates, eg (2π/3, 1, 2) or constants can be used, e.g. (a, b, c).

To **move** a point, either drag it with the mouse (when it will follow the location rule above), or, better use the ...

Arrow keys:	Left//Right	adjusts the 'x' value
	Up/Down	adjusts the 'y' value
	PgUp/PgDn	adjusts the 'z' value.

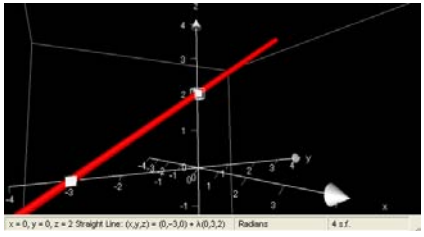
The incremental step is also controlled as in 2D, and relative to the grid spacing: 0.1, otherwise hold CTRL: 0.01 and SHIFT-CTRL: 0.001.

1 In the Standard Level, all these increments are multiplied by 10 when the integer snap is on.

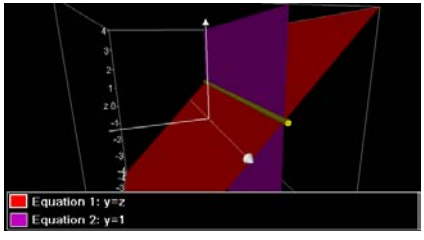


With a single point selected, use the option "Coordinate Box" to show clearly what the coordinates are.

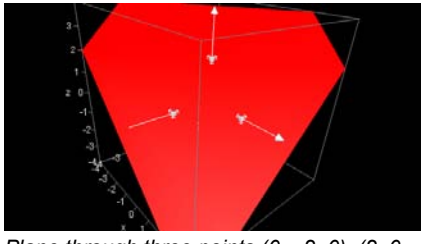
3D: Straight Lines and Planes



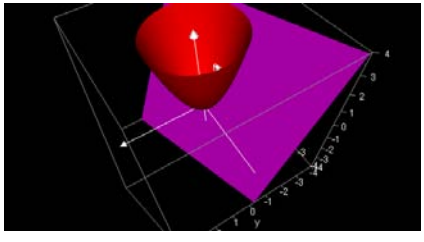
The Straight line (0, −3, 0) and (0, 0, 2): [x, y, z] = [0, −3, 0] + λ[0, 3, 2]



Intersection of two planes: [x,y,z] = [0,1,1] + λ[1,0,0]; Angle between planes = 45°



Plane through three points (0, –2, 0), (2, 0, 0) and (0, 0, 2) : 0.5x – 0.5y + 0.5z = 1



Tangent plane to a paraboloid. All results obtained from "Page" => "Copy Status Bar".

STRAIGHT LINES in 3D

All lines are given in the Status Bar as: [x, y, z] = [a, b, c] + λ[f, g, h]

ONE POINT selected:
♦ **Line // x-axis** ♦ **Line // y-axis** ♦ **Line // z-axis**

TWO POINTS selected:
♦ **Straight Line**
♦ **Line Segment** Distance given
♦ **Gradient** Δx, Δy, Δz given

Vector Line [x, y, z] = [a, b, c] + λ[f, g, h]
A straight line can also be entered directly in vector form

ONE POINT attached to a PLANE or SURFACE
♦ **Normal Line**

TWO PLANES selected
♦ **Intersection line**

ONE POINT and a VECTOR selected
♦ **Line from vector**

PLANES in 3D

Planes are given in the Status Bar as: ax + by + cz = 1 or in vector form [x, y, z] = [a, b, c] + λ[f, g, h] + μ[l, m, n], *as appropriate*.

ONE POINT selected:

♦ **Plane ⊥ x-axis, or ⊥ y-axis, or ⊥ z-axis**

THREE POINTS selected, or
ONE POINT and a STRAIGHT LINE or VECTOR selected
♦ **Plane** (vector ***//*** the plane)

ONE POINT and a VECTOR selected
♦ **Plane from Vector** (vector ⊥ the plane)

ONE POINT and TWO VECTORS selected
♦ **Plane** [x, y, z] = [a, b, c] + λ[f, g, h] + μ[l, m, n]

Vector plane can be entered directly in the form
[x, y, z] = [a, b, c] + λ[f, g, h] + μ[l, m, n]

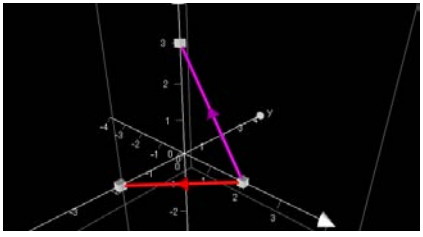
Equations of planes can be entered directly
eg: x = k, y = k, x + y = k, ax + by + cz = d

ONE POINT attached to a STRAIGHT LINE selected:
♦ **Perpendicular Plane**

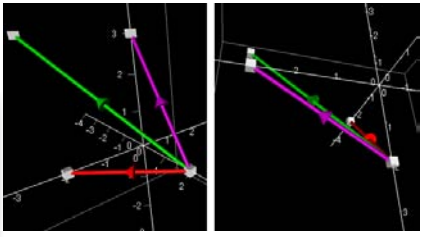
ONE POINT attached to a z = f(x, y) SURFACE selected:
♦ **Tangent Plane (small or full)**

ONE POINT and a VECTOR selected
♦ **Plane from vector (perpendicular to the vector)**

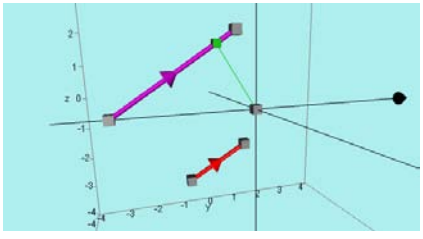
3D: Vectors



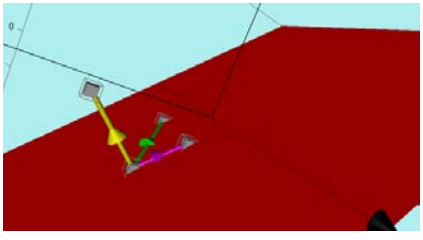
As in 2D, there are two vector definitions: enter the coordinates, or join two points.



The sum of two vectors: use of the drag zoom to show they are in the same plane.



An example of a scalar multiple of a vector, and the closest distance from a point.



Two vector from three (draggable) points attached to a plane, at their cross product.

VECTORS in 3D

VECTOR CREATED FROM A SINGLE POINT
As in 2D, a 3D vector must have a point to 'sit' on, so to create a vector you must have a point selected. Use the right-click option "Vector" and enter the Cartesian coordinates [x, y, z], or the Polar Coordinates [r, θ, ϕ]. If constants are used here, these can be altered using the Constant Controller.

The vector can be moved by dragging the base point.

VECTOR CREATED FROM TWO POINTS
Alternatively, a vector can be created between two selected points, making it moveable at either end.

A vector consists of three objects: base point, end point and stem (with an arrow at the mid point). A vector is selected by clicking on its stem.

With ONE VECTOR [a, b, c] and a POINT [l, m, n] selected, a new vector can be created based on the point as follows:

- **Negative vector** [–a, –b, –c]
- **Copy vector** [a, b, c]
- **Unit vector** [a/r, b/r, c/r], $r = \sqrt{(a^2 + b^2 + c^2)}$
- **Multiply vector (enter factor)** [ka, kb, kc]
  This creates a scalar multiple of the original vector. 'k' can animated.
- **Line from vector** [x, y, z] = [l, m, n] + λ[a, b, c]
- **Closest point, and distance** *on the line of the vector*

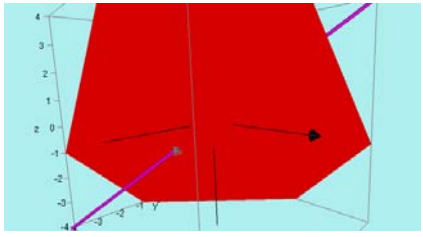
TWO VECTORS [a, b, c] and [l, m, n] selected,
- **Angle between two vectors (*)** *degrees or radians*
- **Scalar (or Dot) Product (*)** *al + bm + cn*

(*) *given in the Results Box*

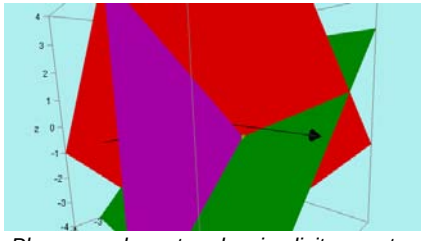
TWO VECTORS [a, b, c] and [l, m, n]
and a POINT [p, q, r] selected, a new vector can be created based on the point as follows:

- **Add vectors** [a + l, b + m, c + n]
- **Subtract vectors** [a – l, b – m, c – n]
- **Cross Product**

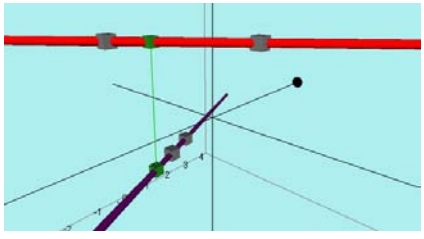
With a POINT and a PLANE or z = f(x,y) SURFACE selected
- **Normal Unit Vector**
- **Normal Line**



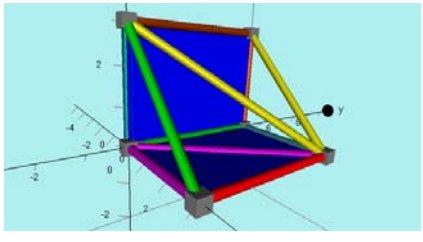
A point is attached to the intersection between a vector line and a plane.



Planes can be entered as implicit or vector equations, or created from 3 points.



The shortest distance between two lines. Status bar gives the distance and the angle.



A 3D shape made up of line segments. Select any two to find the angle.

CALCULATED POINTS in 3D

As in 2D, *"Calculated Points" are not added to the set of points, and are indicated by a small coloured cube. They can be selected (for deletion), and a point can be attached to a calculated point.*

STRAIGHT LINE and a PLANE selected
♦ **Intersection point**

THREE PLANES selected
♦ **Intersection point**

TWO POINTS selected
♦ **Mid Point**

THREE or more POINTS selected
♦ **Centroid**

CLOSEST DISTANCES in 3D

The Status Bar gives coordinates of the end point(s), the distance, and angle as appropriate.

ONE POINT and a PLANE selected
♦ **Closest point**

ONE POINT and any LINE or VECTOR selected
♦ **Closest point**

TWO STRAIGHT LINES selected
♦ **Closest distance**

CALCULATED ANGLES in 3D

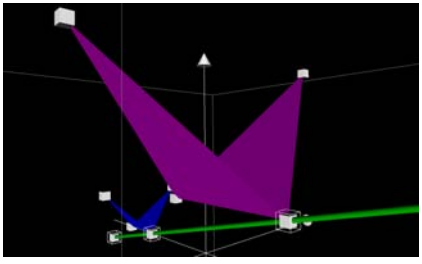
Given the results box, in degrees or radians. A "LINE" here means a Straight Line, Vector or Line Segment.

TWO LINES (or any combination) selected

ONE STRAIGHT LINE and a PLANE selected

TWO PLANES selected

3D: Shapes and Transformations



A 3D shape (blue) created from four points, enlarged through a point to a purple shape.

Transformations by 3D MATRIX

User defined: enter your own elements (by columns); press "Inverse" to change to the elements of the Inverse Matrix.

Presets: The following are available:

Rotation about x-axis

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

Rotation about y-axis

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

Rotation about z-axis

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Enlargement

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

If any of the elements contain constants, these can be controlled by the constant controller.

With the transformed shape selected, use the Animation controller to apply the matrix 'n' times (n = –1 is equivalent to the inverse matrix).

CREATING A SHAPE in 3D

A shape in 3D must necessarily be made up of triangles. To create a finished shape there are several options:

1. Use "Enter Shape", and select one of the Presets (all five of the Pentatonic solids):

♦ **Tetrahedron**	4 points,	4 triangles	or 6 lines
♦ **Octahedron**	6 points,	8 triangles	or 12 lines
♦ **Cube**	8 points,	12 triangles	or 12 lines
♦ **Icosahedron**	16 points,	20 triangles	or 30 lines
♦ **Dodecahedron**	20 points,	36 triangles	or 32 lines

The default is to use filled triangles, to give the shape a solid appearance, but there is the option to show unfilled triangles, or lines only.

2. If you know the coordinates of all the points, use "Enter Shape" and enter the coordinates manually. Then, 3 points at a time, click "Add" to add them to the list of triangles that will make up the shape. Alternatively, select two points at a time and create a list of Lines.

3. Select 3 points and use "Group to Shape". To extend the shape, select a new point and two of the existing shape's points and use "Group to Shape" again, and so on.

TRANSFORMING A SHAPE in 3D

To select a shape, click on one of its solid sides. If it is "Lines" only, click on one of the lines.

ONE SHAPE selected

- ♦ **Rotation about x-axis**
- ♦ **Rotation about y-axis**
- ♦ **Rotation about z-axis**
- ♦ **MATRIX transformation**
- ♦ **Reflection in the plane x = 0**
- ♦ **Reflection in the plane y = 0**
- ♦ **Reflection in the plane z = 0**

ONE SHAPE and ONE POINT selected

- ♦ **Enlargement** Scale factor can be animated

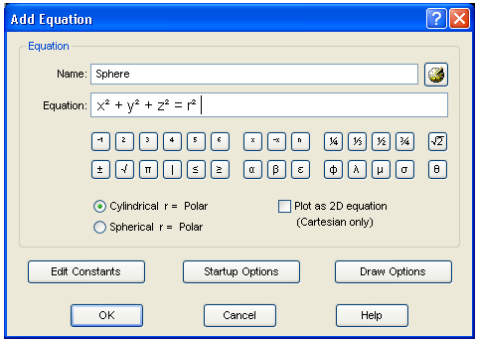
ONE SHAPE and ONE LINE selected

- ♦ **Rotation** Angle can be animated

ONE SHAPE and ONE PLANE selected

- ♦ **Reflection**

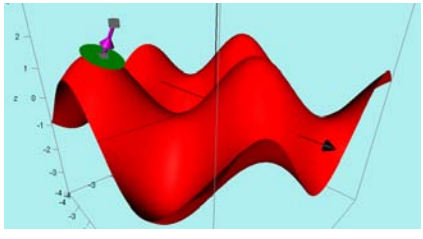
3D: Entering Equations



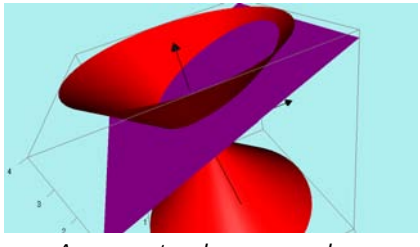
3D Equation entry is very similar to 2D. "zz" enters z². Each equation has its own appropriate "Start-up Options".

*Use "**Plot as 2D**" option when there is no 'z', and you want to have no 'z' plotting at all (eg for Volumes of Revolution).*

***"Draw Options":** set surface colour, and transparency here.*



A z = f(x, y) equation with a point attached, and a tangent plane and normal unit vector.



A cone, entered as r = z, and an intersecting plane, entered as z = ax + b

Autograph interprets the entered equation according to its form:

Surface: EXPLICIT EQUATIONS: z = f(x,y)

e.g.: **Egg box:** z = asinxcosy
Saddle: $z = x^2 - y^2$
Paraboloid: $z = x^2 + y^2$

Start-up Options: you can to set the number of 'x' and 'y' divisions (default 50 x 50) to a max of xy = 200 000 (e.g. 400 x 400). Higher settings will be very slow to plot, and could run out of graphics memory!

Surface: 1st DEGREE IMPLICIT EQUATIONS

These are rearranged internally as z = f(x,y)
e.g.: **Plane:** ax + by + cz = d
this is equivalent to the vector equation:
[x, y, z].[a, b, c] = d, or **r. n = d**

ADDING A POINT TO A SURFACE
A moveable point can be placed on any of the above.
For a given (x, y), 'z' will be calculated to place it on the surface.
The following right-click options are available:

- **Normal Line:** [a, b, c] + λ[l, m, n]
- **Normal Unit Vector:** [a, b, c]
- **Tangent Plane (small or large):** ax + by + cz = 1

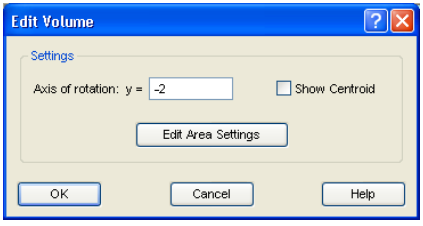
Surface: GENERAL IMPLICIT EQUATIONS

These plots are not interpreted as functions, so points cannot be placed on them. The plotting system used is generally very satisfactory, but it can be less than successful when the plotting involves tight corners. The quality can be improved by increasing the resolution in "**Startup options**" in the Equation Entry box: x-div = 16 is the default. Up to x-div =30 is acceptable.

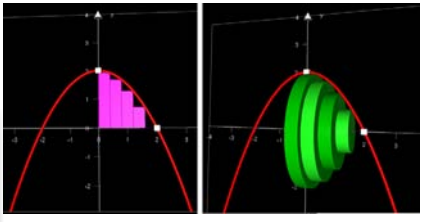
e.g. **Box:** |x| + |y| = r
Schwarz's surface: cosx + cosy + cosz = 0
Cone: $x^2 + y^2 = z^2$ [or cylindrical polar: r = z]
Sphere: $x^2 + y^2 + z^2 = r^2$

Cylinder: $x^2 + y^2 = a^2$ [or cylindrical polar: r = a]
$x^2 + z^2 = a^2$
$y^2 + z^2 = a^2$

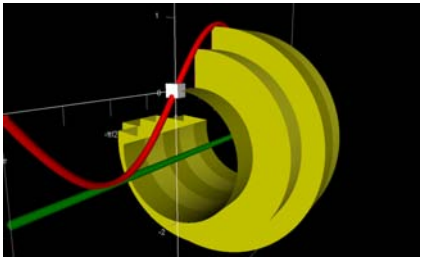
3D: Volume of Revolution



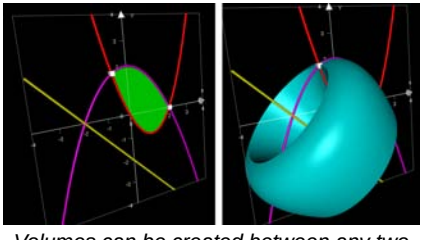
Here you enter the axis of revolution y = ... and you can also edit the Area settings



The area between $2y = 4 - x^2$ and the x-axis is rotated about the x-axis (or any axis entered in the Edit Volume box).



Using Slow Plot, the volume can unfold slowly. The animation controller can also do this manually, or automatically.



Volumes can be created between any two curves and about any straight line. Here Simpson's Rule creates a smooth surface.

Autograph's 3D environment provides a natural setting for the study of volumes of revolution. The sequence to follow is:

1. **Select "x-y Orientation"** on a 3D page

2. **Enter a y = f(x) graph**, and select "Plot as 2D"
e.g.: y = sinx, or $2y = 4 - x^2$

3. **Create an area under the curve**
Either: select the graph or: select two points attached to the graph
Then use the right-click option "Area". This works exactly as it does in 2D with the four options:
Rectangle (+), Rectangle (–), Trapezium Rule, Simpson's Rule
and enter the x-limits for the area (if not already defined by points) and enter the number of divisions (which can be altered dynamically later on).
"Show Centroid": $(\bar{x}, \bar{y})$ for the area as a "calculated" point.

4. "**Slow Plot**": you will see the volume open out slowly.

5. **Select the area**. Use the right-click option **"Find Volume"**.
Enter axis of rotation y = ... (Enter a number or a constant. The default: y = 0, the x-axis).
"Show Centroid": $(\bar{x}, \bar{y}, 0)$ for the volume as a "calculated" point.
Status Bar: shows volume (as multiple of π, and area).

6. Select the volume: use the "**Animation Controller**" to vary
(a) the Volume (from 0 to 2π, or 360°)
(b) the number of divisions
These can be varied manually using the Up/Down arrow keys, or using the "Animate" options.

Using **Simpson's Rule**, the volume will appear very smooth, and the calculation (in the status bar) will be the most accurate.

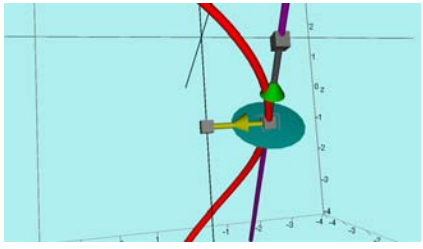
OTHER OPTIONS

♦ **Volume between x = f(y) and a x = k axis**
Everything works the other way round.

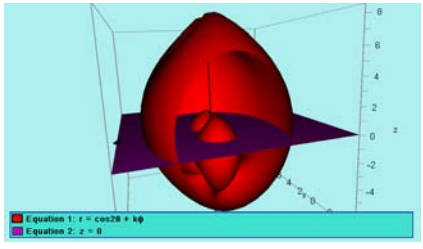
♦ **Volume of an area between two functions**
Create an area as in 2D: select the two functions, or select one point on each curve.

♦ **Volume of revolution of an area about any straight line.**
Select the area and any straight line. The resulting volume is created but sloping lines are not calculated!

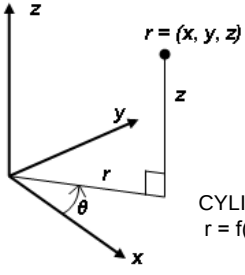
3D: Parametric and Polar Equations



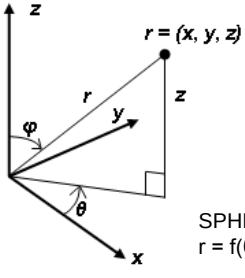
A parametric spiral, with a point attached, and tangent, normal plane, velocity and acceleration vectors created. This image used the camera zoom on the "Drag" mode.



A bit of fun with a spherical coordinate plot,
r = cos2θ + kφ
and its intersection with the plane z = 0.



CYLINDRICAL POLAR:
r = f(θ, z)



SPHERICAL POLAR:
r = f(θ, ϕ)

PARAMETRIC EQUATIONS
x = f(t), y = g(t), z = h(t)

These are all LINES in 3D, each point (x, y, z) depending on the value of the parameter 't'. To make the line thicker (or a different colour), use "Draw Options" in the Equation entry box, or select the drawn line, and use the Line Thickness button. Thick lines in 3D are drawn as cylinders. The range and step for 't' is set in the Start-Up options as follows:

Generally:	–5, 5	step 0.1
Trig functions (radians):	0, 2π	step π/25
Trig functions (degrees):	0, 360°	step 4°

e.g.: **spiral** x = sint, y = cost, z = t
pair of straight lines x = t, y = ±t, z = 0

With a (moveable) POINT attached to a parametric graph, you can find the:

- **Tangent** [a, b, c] + λ[l, m, n]
- **Normal plane (large or small)** ax + by + cz = 1
- **Velocity Vector** [a, b, c]
- **Acceleration Vector** [a, b, c]

CYLINDRICAL POLAR
r = f(θ, z) or r² = f(θ, z) or 1/r = f(θ, z)

In the Equation Entry box, there is the option to select 'r= ..' to be interpreted as cylindrical or spherical coordinates. Any 2D polar equation can also be interpreted as a Cylindrical Polar.

e.g.: **cylinder:** r = 1
triangular prism, box: r = 1
(use θ-step set to 2π/3 or π/2, in "Startup Options")
cone: r = z

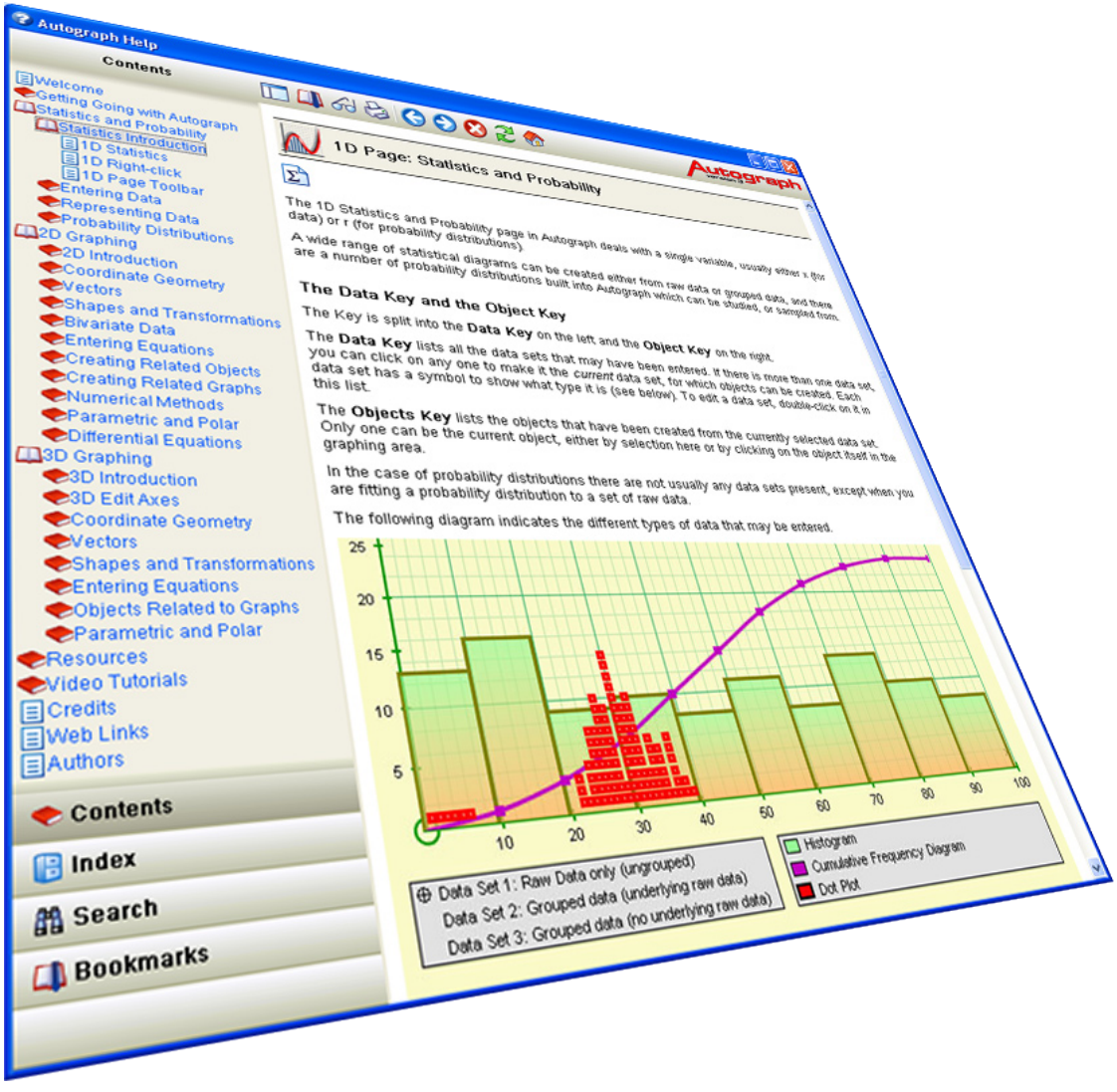
SPHERICAL POLAR
r = f(θ, φ) [ALT-T = θ] [ALT-F=ϕ]

e.g.: **sphere:** r = 1
shell on the shore! r = 1.3^θ sinϕ

Spherical Polar coordinates can also be entered parametrically:
x = f(θ,ϕ), y = f(θ,φ), z = f(θ,ϕ)
e.g.: **torus:** x=cos(θ)(p+qcos(ϕ)), y=sin(θ)(p+qcos(ϕ)), z=qsin(ϕ)

To convert from Spherical Polar to Parametric Form:
r = f(θ,ϕ) => x = rsinϕcosθ, y = rsinϕsinθ, z = rcosϕ

HELP and RESOURCES



Using Autograph Help [F1]

The Autograph Help consists of nearly 300 interlinked pages. Press **F1** any time that Autograph is running, or go to “Help” => “Autograph Help”. Alternatively, the Help file can be found in “Start” => “All Programs” => “Autograph 3.20” => “Help”.

There are four accordian items on the left:
CONTENTS offers an expandible list capable of showing every page in the help system.
SEARCH allows a powerful keyword search of the entire contents.
BOOKMARKS lists your bookmarked searches and topics.
INDEX provides an alphabetical list of key points.

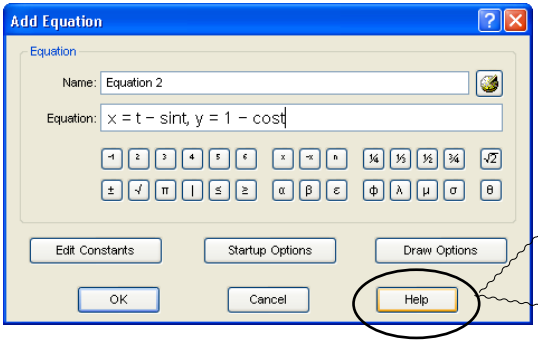
EXAMPLE EQUATIONS
This section lists a number of equations in each of the following categories. Any equation can be selected and copied into the 2D or 3D “Add Equation” dialogue box:

2D Graphing Page

Cartesian y = f(x)	Cartesian x = f(y)
Trigonometric	Inverse Trig
Exponential	Hyperbolic
Implicit => y = f(x)	Conics
General Implicit	Fun Equations
Inequalities	Piecewise
Function Definitions	Evolutes
Parametric Equations	Polar Equations
1st Order Diff. Eqns	2nd Order Diff. Eqns

3D Graphing Page

z = f(x, y)	Implicit equations
Intersections with a cone	Parametric Coordinates
Spherical polar	Cylindrical polar



FORMULAE
This section is a comprehensive list of formulae for school and college mathematics. Any formulae can be selected and pasted (e.g. into Word). All symbols are provided by the “Arial for Autograph” font.

Pure Mathematics

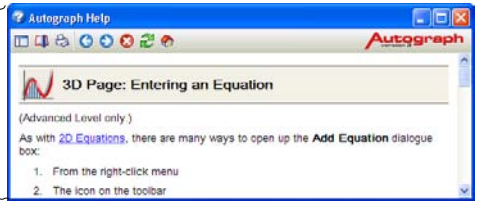
1. Logarithms, Exponentials	2. Trigonometry
3. Radians and Degrees	4. Maclaurin Expansions
5. Differentiation	6. Integration

Probability and Statistics

1. Sample Statistics	2. Population Statistics
3. Selections	4. Discrete Prob. Distr.
5. Continuous Prob. Distr.	6. Central Limit Theorem

Other Formulae
Numerical Methods for finding the area under a curve
Matrix Transformations
Differential Equations: Numerical methods

CONTEXT SENSITIVE HELP
Most Autograph dialogue boxes have a “Help” button. This will take you to the appropriate page in the Help system:



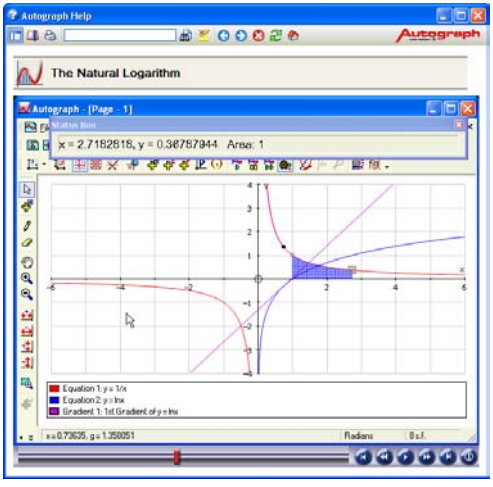
Using Autograph Tutorials [F3]

One of the best ways to get to see what Autograph is capable of is to view the Tutorials.

1. In the Help File
A number of tutorials have been included in the Help application in the final section, and appropriate tutorials are linked from individual help pages. It may be necessary to open out the Help window a bit in order to view them satisfactorily.

2. On the Autograph Web Site

From the Autograph home page, click on the 'Autograph In Action' link for the page containing the contents of the Tutorials.



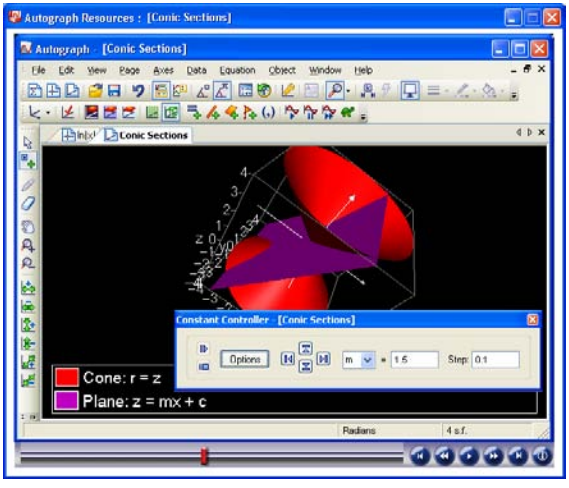
All the tutorials are based on classroom experience. Here the important link between ln|x| and logx can be visualised.

TurboDemo
The tutorials have each been created by recording a series of screen-shots while Autograph is running. The **TurboDemo** application has been used for this, allowing visual effects and explanatory text 'bubbles' to be added.

TurboDemo tutorials give the appearance of a real-time movie, but are in fact a set of discrete images, but with the mouse movements joined up.

The tutorial contents list has been arranged to reflect the two 'levels' of Autograph:
- Standard
- Advanced.

Within those two levels topics are also arranged to match the three 'page' types in Autograph:
- Statistics and Probability
- 2D Graphing
- 3D Graphing



The 3D pages of Autograph are especially effective in tutorials. Here the conic sections are being explored.

All the Autograph tutorials can be controlled by the slider (to move quickly to a particular frame), or by the buttons which give the usual options: back to the start, back one frame, play/pause, forward one frame, forward to the end.

The 'Arial for Autograph Uni' Font

The ARIAL FOR AUTOGRAPH' Unicode font has been specially commissioned from Monotype Imaging (UK) to include as many characters as possible that are useful in school/college level mathematics. These characters are available for use in Autograph, as well as any other application that handles text (e.g. Word).

On the keyboard: | ~

Off keyboard, non-Unicode: ° ± ² ³ ÷ × –

Circumflex: â ĝ ĉ î ĵ ǩ ĥ ŝ ŵ ẑ û ŷ ô ê û

Dots and bars: ẋ ẏ ṙ θ̇ v̇ ṡ ψ̇ ż ẍ ÿ r̈ ö x̄ ȳ X̄ Ȳ r̄

Superscripts: –1 2 3 4 5 6 7 8 9 + – n x –x y –y r

Subscripts: 1 2 3 4 5 6 7 8 9 + – n a i ( ) = 0

Greek Alphabet: Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω
α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω

Fractions: ½ ⅔ ⅓ ⅔ ¼ ¾ ⅕ ⅖ ⅗ ⅘ ⅙ ⅚ ⅛ ⅜ ⅝ ⅞

Miscellaneous: € ℓ Å ℂ ℕ ℙ ℚ ℝ ℤ

Arrows: ← ↑ → ↓ ↔ ↕ ↖ ↗ ↘ ↙ ↞ ↟ ↠ ↡ ↵
↶ ↷ ↹ ↺ ↻ ↛ ⇐ ⇑ ⇒ ⇓ ⇔ ⇕ ⇖ ⇗ ⇘ ⇙

Mathematical symbols: ∧ ∨ ⊂ ⊃ ∩ ∪ ⊄ ⊅ ⊆ ⊇ ∈ ∉ ∅ ≃ ≈ ≙ ≠ ≡ ≤ ≥
∀ ∃ ∂ Δ ∏ ∑ ∫ ∝ ∞ ∡ ⊥ − ∴ ∵ ∶ √ ∛ √2 √3

Miscellaneous symbols: † ‡ • … ‘ ’ “ ” ⊗ ⊙ ○ ☼ ☹ ☺ ♀ ♂ ⌘ ⏲ ⧗ ⌨
☁ ☂ ☐ ☑ ☒ ✓ ✗ ❑ ❄ ❖ ✍ ✎ ✑ ☝ ☞ ✌ ✂ ✄ ✈ ✉

Chess and playing cards: ♔ ♕ ♖ ♗ ♘ ♙ ♚ ♛ ♜ ♝ ♞ ♟ ♠ ♥ ♦ ♣ ♤ ♡ ♢ ♧

Music: ♩ ♪ ♫ ♬ ♭ ♯ ♮

Large brackets : ( ) [ ] { } | | ∫

Use of Unicode Fonts generally

All the above are in their standard UNICODE positions, so will work in all FULL Unicode fonts, e.g. Arial Unicode MS, EXCEPT the large brackets and following, which have been added as Arial for Autograph ***Private Use Characters****:*

–1 x –x –y a i n ẟ ǩ ṅ t̂ ô ê û p̂ θ̇ ψ̇ v̇ √2 √3 r̄ x̄ X̄ Ȳ ⅔

Expressions which do not involve these 'Private Use' characters will appear correctly rendered using any standard Unicode font in any document, email, web site, etc. Such expressions can be freely copied into Autograph, or pasted out.

Finding Useful Characters

A number of mathematical symbols exist in the early pre-256 font positions, which can be entered using ALT and their ASCII decimal code (on the key-pad),
eg 'x' followed by ALT 0178 will enter x^2:

0128	0149	0150	0176	0177	0178
€	•	–	°	±	²
0179	0188	0189	0190	0215	0247
³	¼	½	¾	×	÷

When running *Autograph*, a number of useful characters can be entered any time using the ALT key, eg 'x' followed by ALT 1 will enter x^{-1}:

ALT	1	2	3	4	5	6	7	8	9	X	Y	N
Indices	x^{-1}	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9	e^x	e^y	x^n

ALT	A	B	E	F	G	L	M	S	P	T
Greek	α	β	ε	ϕ	γ	λ	μ	σ	π	θ

'PI' also enters π

ALT	0	-	+	H	<	>	R	O	C
Maths	∞	–	±	½	≤	≥	√	°	∛

'<=', '>=' also enter ≤, ≥

Using Character Map

The complete *Arial for Autograph Unicode* font can be explored using the Windows utility *Character Map*. This can usually be found in
Start => Programs => Accessories => System Tools

You can use 'Right-click drag' to create a short-cut to this, eg on the desktop, or in the Start Menu.

After you have opened *Character Map*:
- Find the font *Arial for Autograph Uni* in the font list
- Click on any character you want to copy
- Click select adds it to the list of Characters to copy
- press Copy: the characters are now on the clipboard.

Using *Character Map* to explore the font:
- Tick Advanced View
- set Character Set to Unicode
- Group by Unicode Subrange

The Autograph **on-screen keyboard** will also deliver a wide range of mathematical symbols into whatever application is running (eg Word).

To launch the On-screen Keyboard withough running Autograph, go to Start => Programs => Autograph 3.20 => Keyboard.

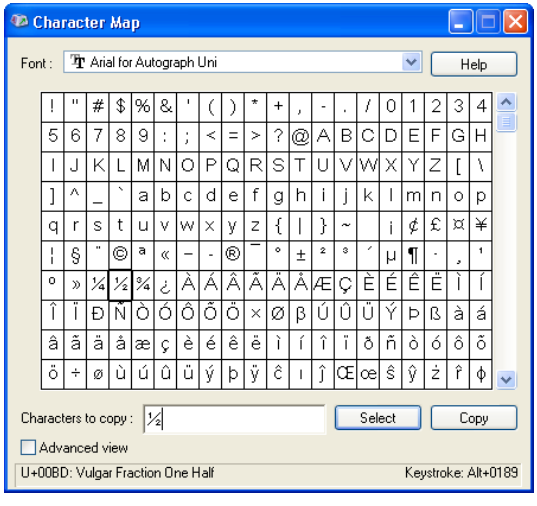
Since most of the common mathematical symbols are in their standard Unicode positions, delivering the symbols does not require the *Arial for Autogrpah Uni* font, unless you are wanting to use symbols from the Private Use area.

MINUS SIGN:

It is very important that mathematical expressions use a proper MINUS sign '**–**' and not the hyphen '-'. This can always be added to any document using ALT 0150.

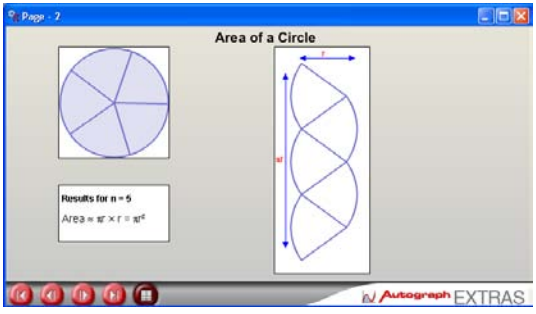
Within Autograph itself, the hyphen enters a MINUS when you are typing in equations.

Another source for the MINUS is on the Autograph onscreen Keyboard, using the "Data Entry" panel.



The Autograph Extras

In the "**File**" menu => "**New Extras Page**", there are SIX interactive flash demonstrations, each of which will open in a new, sizeable Autograph page. They are also found in the "Autograph Resources" application [F3].

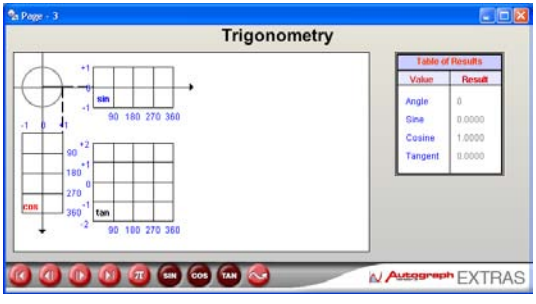


AREA OF CIRCLE

An animation of the limit of n sectors of a unit circle placed back to back. In the limit, this illustrates that the area of a circle = πr^2.

Control buttons:

- Restart
- Increment 'n' down by 1
- Increment 'n' up by 1
- Go to 'n' = 360
- Show information

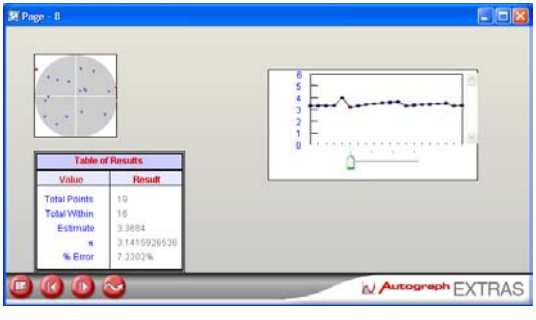


TRIGONOMETRY

An animation showing the link between the unit circle and the three (circular) trigonometric functions.

Control buttons:

- Restart
- Increment 'θ' down by 15° (can go beyond 360°)
- Increment 'θ' up by 15° (can go below 0°)
- Go to 'θ' = 360
- Switch Degrees/Radians (starts in Degrees)
- SIN, COS, TAN - on/off
- Automatic demonstration

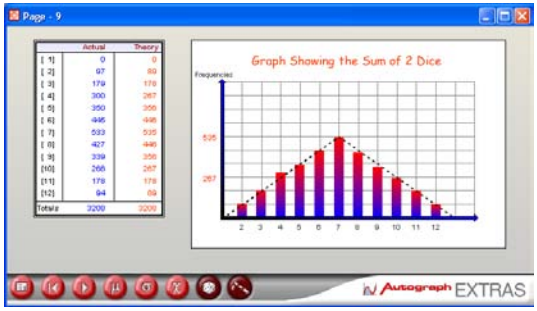


MONTE CARLO - π

This simulation of random points in a unit circle inside a square of side 2, leading to:
$\pi \approx$ (Points within the circle/Total Points)*4
Opening option: Auto (up to n = 10,000) and fixed.

Control buttons:

- Opening option again
- Restart the simulation
- Increment 'n' a few up (if paused)
- Pause/resume
- Slider: 4 levels of zoom



DICE SIMULATION options:
1 Die - Sum of 2 Dice - Difference of 2 Dice
Maximum of 2 Dice - Sum of 'n' Dice (max 25)

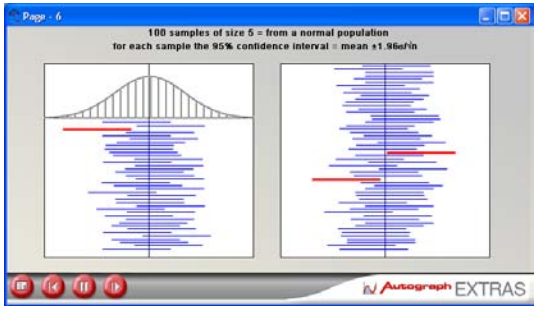
DICE SIMULATION

A simulation of throwing dice.
Opening option: Auto / Fixed number (max 9999)

Control buttons: (Starts paused)

- Opening Options
- Restart
- Start / Pause
- Display mean, μ
- Display SD, σ
- Display χ^2
- Display Actual frequencies
- Display Theoretical Frequencies

TWO FURTHER EXTRAS for the Advanced Level only



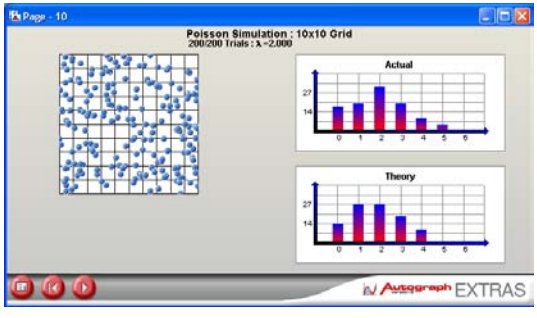
Confidence level = 90%: Interval $\bar{x} \pm 1.64\sigma/\sqrt{n}$
Confidence level = 95%: Interval $\bar{x} \pm 1.96\sigma/\sqrt{n}$
Confidence level = 99%: Interval $\bar{x} \pm 2.58\sigma/\sqrt{n}$

CONFIDENCE INTERVALS

For each of 100 samples of size 'n' (default = 5, max 30), the confidence intervals are shown, and those not including the population mean (zero) are highlighted.
Choice of Confidence levels: 90%, 95%, 99%.

Control buttons: (Starts paused)

- Opening Options
- Restart
- Start / Pause
- Increment the simulation



POISSON GRID

A simulation of 'n' (default 200, max 1000) random points falling on a square 10 x 10 grid. Frequencies are counted for the squares that contain 0, 1, 2, 3, ... points. The distribution is compared with the theoretical Poisson distribution, with $\lambda = n/100$

Control buttons: (Starts paused)

- Opening Options
- Restart
- Start / Pause

Statistics Formulae

♦ SAMPLE STATISTICS

Size of sample: $n = \Sigma f$

Mean: $\bar{x} = (1/n)\Sigma(fx)$

Variance: $s^2(x) = (1/n)\Sigma f(x - \bar{x})^2 = (1/n)\Sigma fx^2 - \bar{x}^2$

Coding: $y = ax + b$, $\bar{y} = a\bar{x} + b$, $s^2(y) = a^2 s^2(x)$

Covariance:

$\text{cov}(xy) = (1/n)\Sigma(x - \bar{x})(y - \bar{y}) = (1/n)\Sigma xy - \bar{x}\bar{y}$

Product Moment Correlation Coefficient, PMCC:

$r = \text{cov}(xy)/(s(x)s(y))$

Least-Squares Regression Line, y-on-x:

$y - \bar{y} = \text{cov}(x,y)/s^2(x).(x - \bar{x})$

OR $y = a + bx$,

where: $\Sigma y = na + b\Sigma x$ and $\Sigma xy = a\Sigma x + b\Sigma x^2$

Least-Squares Regression Line, x-on-y:

$x - \bar{x} = \text{cov}(x,y)/s^2(y).(y - \bar{y})$

Spearman's Rank Correlation Coefficient:

$r(s) = 1 - (6\Sigma d^2)/(n(n^2 - 1))$

where: d = difference between the rankings

♦ POPULATION STATISITICS

Expectation:

$\mu = E[X] = \Sigma px$ where $p = P(X=x)$

Variance: $\sigma^2 = \text{Var}(X) = E[X - \mu]^2 = E[X^2] - \mu^2$

$= \Sigma p(X - \mu)^2 = \Sigma px^2 - \mu^2$

Coding: $E[ax + b] = aE[x]+b$ $\text{Var}(aX+b) = a^2\text{Var}(X)$

Selections and Combinations

$nPr = n!/(n - r)!$

$nCr = nCr/r! = n(n-1)(n-2)\ldots(n-r+1)/r!$

♦ DISCRETE PROBABILITY DISTRIBUTIONS

Rectangular:

$X \sim R(a, b)$ $r = a, a+1, \ldots, b$

$P(X = r) = 1/(b - a + 1)$

Mean, $\mu = (a + b)/2$

Variance, $\sigma^2 = (b - a)(b - a + 2)/12$

Binomial:

$X \sim B(n,p)$ $q = 1 - p$

$P(X = r) = nCr\ p^r\ q^{(n-r)}$ $r = 0,1,2,3, \ldots, n$

Mean, $\mu = np$

Variance, $\sigma^2 = npq$

Poisson:

$X \sim Poi(\lambda)$ $\lambda > 0$

$P(X = r) = e^{(-\lambda)}.\lambda^r/r!$ $r = 0,1,2,3, \ldots$

Mean, $\mu = \lambda$

Variance, $\sigma^2 = \lambda$

Geometric:

$X \sim G(p)$ $q = 1 - p$

$P(X=p) = q^{(r-1)}.p$ $r = 1, 2, 3, \ldots$

Mean, $\mu = 1/p$ Variance, $\sigma^2 = q/p^2$

User-defined discrete probability distribution:

Mean, $\mu = \Sigma r.P(x=r)$ $r = 0,1,2,3 \ldots$

Variance, $\sigma^2 = \Sigma r^2.P(X=r) - \mu^2$

♦ CONTINUOUS PROBABILITY FUNCTIONS

Uniform:

$X \sim U(a,b)$ $a \le x \le b$

Mean, $\mu = (a + b)/2$

Variance, $\sigma^2 = (a - b)^2/12$

Normal: $X \sim N(\mu, \sigma^2)$

pdf: $f(x) = 1/(\sigma\sqrt{(2\pi)}).e^{(-½((x - \mu)/\sigma)^2)}$

Mean = μ Variance = σ^2

Normal: $Z \sim N(0, 1)$

pdf: $\phi(z) = 1/\sqrt{(2\pi)}.e^{(-½z^2)}$

Mean = 0 Variance = 1

Normal approximations (for suitable n, p and λ):

$B(n, p) \simeq N(np, npq)$

$Poi(\lambda) \simeq N(\lambda, \lambda)$

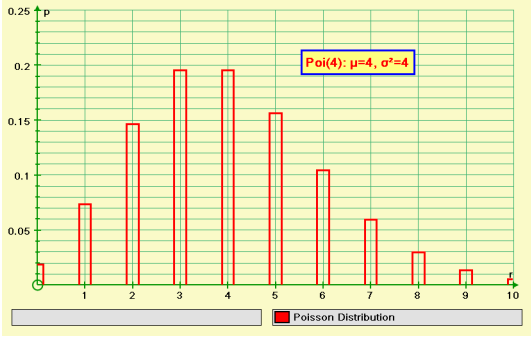
User Defined pdf: $X \sim f(x)$

Mean, $\mu = \int x.f(x)\,dx$

Variance $= \int x^2.f(x)\,dx - \mu^2$

♦ Central Limit Theorem

For samples of size n drawn from a distribution with mean, μ and finite variance, σ^2, the distribution of the sample means tends to $N(\mu, \sigma^2/n)$ as n increases.



♦ 1ST ORDER DIFFERENTIAL EQUATIONS

Equations can be entered implicitly and rearranged as
$y' = f(x,y)$
We enter the initial conditions: (x1, y1) for which the gradient is now known:
$g1 = f(x1, y1)$.
We need to find a reliable estimate for the next point (x2, y2) using a step h. We have:
$x2 \simeq x1 + h$ and $y2 \simeq x1 + h.g1$
Hence: $g2 \simeq f(x2, y2)$

A better estimate (Runge−Kutta) is:
$y2 \simeq y1 + h.(g1 + g2)/2$
Set h too large (inaccurate) and too small (slow).

♦ 2ND ORDER DIFFERENTIAL EQUATIONS

Equations can be entered implicitly and rearranged as
$y'' = f(x, y, y')$
We enter the initial conditions: (x1, y1) and y'1
We need to find a reliable estimate for the next point (x2, y2) using a step h.
We have $x2 \simeq x2 + h$
and $y''1 \simeq f(x1, y1, y'1)$
and $y2 \simeq y1 + h.y'1 + ½h^2.y''1$
[Taylor's Series, 1st 2 terms]
Hence: $y'2 \simeq y'1 + h.y''1$
and $y''2 = f(x2, y2, y'2)$

A better estimate for y2 is (Runge−Kutta−Butler):
$y2 \simeq y1 + h.(y'1 + y'2)/2 + ½h^2.(y''1 + y''2)/2$

♦ 1ST ORDER PARAMETRIC D.E.

When implemented, equations will be entered in parametric form: x' = f(x, y, t), y' = g(x, y, t) [or the other way round]. We have to enter the initial conditions: (x1, y1) at t = t1 for which the two gradient functions are known:
$x'1 \simeq f(x1, y1, t1)$
$y'1 \simeq g(x1, y1, t1)$
We have $t2 \simeq t1 + h$
and $x2 \simeq x1 + hx'1$, $y2 \simeq y1 + hy'1$
Hence: $x'2 \simeq f(x2, y2, t2)$
$y'2 \simeq g(x2, y2, t2)$

A better estimate for (x2, y2) is (Runge−Kutta):
$x2 \simeq x1 + h(x'1 + x'2)/2$
$y2 \simeq y1 + h(y'1 + y'2)/2$

♦ TRAPEZIUM RULE

h = (b − a)/n with n strips x1 = x0 + h
Area $\simeq$ (h/2)(y0 + 2(y1 + y2 + . . .) + yn)
Volume $\simeq$ Sum of (πh/3)(([f1(x0)² + [f1(x0)] [f1(x1)] + [f1(x1)]²) − ([f2(x0)² + [f2(x0)] [f2(x1)] + [f2(x1)]²))
for x0 from xmin to xmax − h

♦ SIMPSON'S RULE

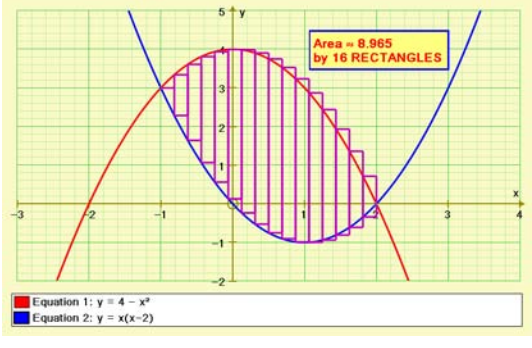
h = (b − a)/n with n strips (n, even) x1 = x0 + h
Area $\simeq$ (h/3)(y0 + 4y1 + 2y2 + 4y3 + . . . + yn)
Volume $\simeq$ Sum of (π(h/2)/3) (([f1(x0)² + 4[f1((x0+x1)/2)²] + [f1(x1)]²) − ([f2(x0)]² + 4[f2((x0+x1)/2)] + [f2(x1)]²))
for x0 from xmin to xmax − h

♦ CENTROID of area under y = f(x)

$A = \int y\,dx$ $\quad \bar{x}.A = \int x.y\,dx$ $\quad \bar{y}.A = \int ½ y^2\,dx$
NOTE: this method will break down if any of the area is of different sign.

♦ MACLAURIN EXPANSIONS

$e^x =$	$1 + x + x^2/2! + x^3/3! + \ldots$	[all x]
$\ln(1 + x) =$	$x - x^2/2 + x^3/3 - x^4/4 + \ldots$	$[-1 < x \le 1]$
$(1 + x)^{-1} =$	$1 + nx + n(n-1)x^2/2! + \ldots$	$[\|x\| \le 1]$
$(1 - x)^{-1} =$	$1 + x + x^2 + x^3 + x^4 + \ldots$	$[\|x\| \le 1]$
$\sin x =$	$x - x^3/3! + x^5/5! - \ldots$	[all x]
$\cos x =$	$1 - x^2/2! + x^4/4! - \ldots$	[all x]
$\arctan x =$	$x - x^3/3 + x^5/5 - \ldots$	$[-1 \le x \le 1]$
$\sinh x =$	$x + x^3/3! + x^5/5! + \ldots$	[all x]
$\cosh x =$	$1 + x^2/2! + x^4/4! + \ldots$	[all x]
$\operatorname{artanh} x =$	$x + x^3/3 + x^5/5 + \ldots$	$[-1 < x < 1]$



Pure Mathematics Formulae

♦ LOGARITHMS AND EXPONENTIALS

$y = e^x \Rightarrow x = \ln y$

$x = e^{\wedge}(\ln x)$

$\ln(e^x) = x$

$\log_a x = \log_b x / \log_b a$

So to enter $\log_b(x)$, use log(x)/log(b)

♦ TRIGONOMETRY

$\cos^2\theta + \sin^2\theta = 1$

$\sec^2\theta = 1 + \tan^2\theta$

$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\tan 2\theta = 2\tan\theta / (1 - \tan^2\theta)$

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$

$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

$\tan(\theta + \phi) = (\tan\theta + \tan\phi)/(1 - \tan\theta\tan\phi)$

$\tan(\theta - \phi) = (\tan\theta - \tan\phi)/(1 + \tan\theta\tan\phi)$

$\sin A + \sin B = 2\sin\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$

$\sin A - \sin B = 2\cos\frac{1}{2}(A + B)\sin\frac{1}{2}(A - B)$

$\cos A + \cos B = 2\cos\frac{1}{2}(A + B)\cos\frac{1}{2}(A - B)$

$\cos A - \cos B = -2\sin\frac{1}{2}(A + B)\sin\frac{1}{2}(A - B)$

$\cos x = \sin(90 - x)$

$\operatorname{cosec} x = \sec(90 - x)$

$\cot x = \tan(90 - x)$

$\sin^{-1}x + \cos^{-1}x = 90°$, etc

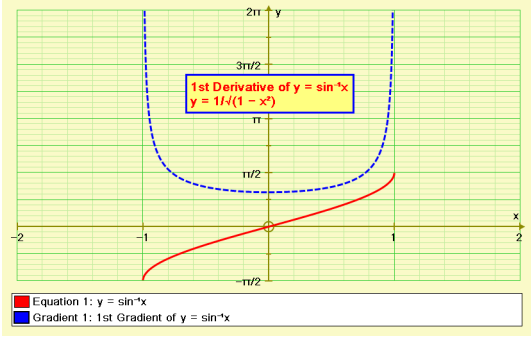
$\operatorname{cosec}^{-1}(x/a) = \sin^{-1}(a/x)$, etc

♦ RADIANS AND DEGREES

Rad:	**0**	**π/6**	**π/4**	**π/3**	**π/2**	**π**
Deg:	0°	30°	45°	60°	90°	180°
sinθ:	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0
cosθ:	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	−1
tanθ:	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0

♦ DIFFERENTIATION

f(x)	f'(x)
x^n	$nx^{\wedge}(n-1)$
e^x	e^x
$\ln x$	$1/x$
a^x	$a^x.\ln a$
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k.\sin(kx)$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\arcsin x$	$1/\sqrt{(1 - x^2)}$
$\arccos x$	$-1/\sqrt{(1 - x^2)}$
$\arctan x$	$1/\sqrt{(1 + x^2)}$
$\operatorname{arcsec} x$	$1/(\sqrt{(x^2 - 1)}.\lvert x\rvert)$
$\operatorname{arccosec} x$	$-1/(\sqrt{(x^2 - 1)}.\lvert x\rvert)$
$\operatorname{arccot} x$	$-1/(x^2 + 1)$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\coth x$	$-\operatorname{cosech}^2 x$
$\operatorname{arsinh} x$	$1/\sqrt{(1 + x^2)}$
$\operatorname{arcosh} x$	$1/\sqrt{(1 - x^2)}$
$\operatorname{artanh} x$	$1/(1 - x^2)$
$\operatorname{arsech} x$	$-1/(x\sqrt{(1 - x^2)})$
$\operatorname{arcosech} x$	$-1/(\lvert x\rvert\sqrt{(1 + x^2)})$
$\operatorname{arcoth} x$	$1/(1 - x^2)$



♦ INTEGRATION

f(x)	∫f(x) dx [+ c, constant]
x^n	x^(n+1)/(n+1) [n ≠ 1]
1/x	ln|x|
e^x	e^x
a^x	a^x/lna
lnx	x.ln|x| − x
sin(kx)	−(1/k)cos(kx)
cos(kx)	(1/k)sin(kx)
tanx	ln|secx|
secx	ln|secx + tanx| = ln|tan(½x+¼π)|
cosecx	ln|tan½x|
cotx	ln|sinx|
$\sin^{-1}x$	x.$\sin^{-1}$x + √(1 − x²)
$\cos^{-1}x$	x.$\cos^{-1}$x − √(1 − x²)
$\tan^{-1}x$	x.$\tan^{-1}$x − ½ln|1 + x²|
$\sec^{-1}x$	x.$\sec^{-1}$x − ln|x + √(x² − 1)|
$\text{cosec}^{-1}x$	x.cosec^{-1}x + ln|x + √(x² − 1)|
$\cot^{-1}x$	x.$\cot^{-1}$x + ½ln|1 + x²|
1/√(a² − x²)	arcsin(x/a) [|x|< a]
1/(a² + x²)	(1/a)arctan(x/a)
1/(a² − x²)	(1/a)artanh(x/a)
	= 1/(2a)ln|(a+x)/(a−x)|
1/√(x² + a²)	arsinh(x/a)
	= ln((x + √ (x² + a²))) [a > 0]
1/√(x² − a²)	arcosh(x/a)
	= ln((x + √ (x² − a²))) [x ≥ a]
e^(ax) sin(bx)	e^(ax)/(a²+b²) (asinbx − bcosbx)
e^(ax) cos(bx)	e^(ax)/(a²+b²) (acosbx + bsinbx)
sinh(x)	cosh(x)
cosh(x)	sinh(x)
tanh(x)	lncosh(x)
sech(x)	arctan (sinh x)
cosech(x)	ln |tanh(x/2)|
coth(x)	ln |sinh x|
arsinh(x)	x arsinh(x) − √(x² + 1)
arcosh(x)	x arcosh(x) − √(x² − 1)
artanh(x)	x artanh(x) + ½ln(1 − x²)
arsech(x)	x arsech(x) + arsinh(x)
arcosech(x)	x arcosech(x) + arsinh(x)
arcoth(x)	x arcoth(x) + ½ln(x² − 1)

The Chain Rule in reverse:
∫ du/dx.f(u) dx = ∫ f(u) du

Integration by parts:
∫uv dx = v.I − ∫ I.dv/dx dx where I = ∫udx

Chain rule: y = f(u) dy/dx = dy/du.du/dx
Product rule: y = uv dy/dx = v.du/dx + u.dv/dx
Quotient rule: y = u/v dy/dx = (v.du/dx − u.dv/dx)/v²

♦ 3D FORMULAE

Equation of a plane:

r.n = d
[x, y, z].[a, b, c] = d
ax + by + cz = d
[x, y, z] = [a, b, c] + λ[u, v, w] + μ[l, m, n]

Equation of line:

[x, y, z] = [a, b, c] + λ[u, v, w]

Scalar Product:

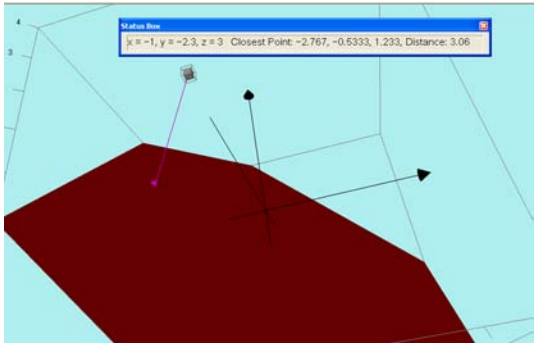
u. v = |**u**|.|**v**|.cosθ

Vector Product:

u x **v** = |**u**|.|**v**|.sinθ.n̂

Distance from point (α, β, γ)
to plane ax + by + cz + d = 0
= |aα + bβ + cγ + d|/√(a² + b² + c²)

Volume:
about x-axis = ∫πy² dx
about y-axis = ∫πx² dy



Example Equations

♦ CARTESIAN y=f(x)

$y = 2x - 3$
$y = mx + c$
$y = (x - a)(x - b)$
$y = x/(3 \pm x)$
$y = x \pm \sqrt{(3 + x)}$
$y = 27/(x^2 + 9)$
$y = 2x/(x^2 + 1)$
$y = (x^2 - 1)/x/(x - 2)$
$y = (x^2 + 1)/(x^2 - 1)$
$y = (1 - x^2)/(1 + x^2)$
$y = ax^2 + bx + c$

then: $x = (-b \pm \sqrt{(b^2 - 4ac)})/(2a)$
and: $x = -b/(2a)$

♦ CARTESIAN x=f(y)

$x = y^2 - 4$
$x = (y - 2)(y + 5)$
$x = \sin y$
$x = 2$
$x = \ln|y|$
$x^2 = \sin y$
$x^3 = \sin y$

♦ TRIGONOMETRIC

$y = \sin^2 x + \cos^2 x$
$y = \cos 2x$
$y = \sin^2 x$
$y = \cos^2 x$
$y = |\sin kx|$
$y = a\sin(nx + \theta)$
$y = c \sin(nx) + d \cos(nx)$
$y = \sin 20x + \sin(nx)$
$y = 2 \sin(\frac{1}{2}(20 + n)x) \cos(\frac{1}{2}(20 - n)x)$
$y = \tan x$
$y = \sec x$
$y = \operatorname{cosec} x$
$y = \cot x$

♦ INVERSE TRIG *Use ALT-1 to enter the symbol* $^{-1}$.

$y = \sin^{-1}x = \arcsin x$
$y = \cos^{-1}x = \arccos x$
$y = \tan^{-1}x = \arctan x$
$y = \sec^{-1}x = \operatorname{arcsec} x$
$y = \operatorname{cosec}^{-1}x = \operatorname{arccosec} x$
$y = \cot^{-1}x = \operatorname{arccot} x$
$x = \sin y$
$y = \sin^{-1}x + \cos^{-1}x$

♦ EXPONENTIAL

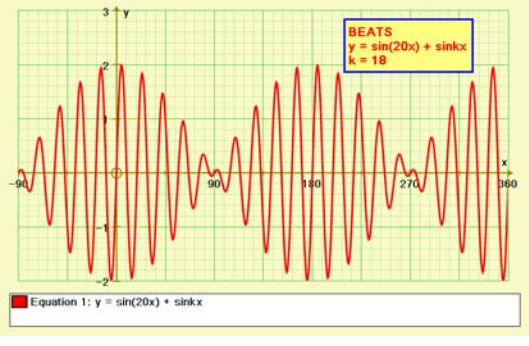
$y = a^x$
$y = x^n$
$y = e^{-x}$
$x = \ln y$
$y = ae^{(-bx)}\sin(cx + d)$
$y = e^x/(1 - e^{-x})$
$y = e^{(-x^2)}$
$y = 1/\sqrt{(2\pi)}e^{(-\frac{1}{2}x^2)}$
$y = \ln|x|$
$y = \ln|(1 - x)/(1 + x)|$
$y = \log x$
$y = \ln x/x$

♦ HYPERBOLIC

Use ALT-1 to enter the symbol '$^{-1}$'.

$y = \sinh x = (e^x - e^{-x})/2$
$y = \cosh x = (e^x + e^{-x})/2$
$y = \tanh x = \sinh x/\cosh x$
$y = \operatorname{sech} x = 1/\cosh x$
$y = \operatorname{cosech} x = 1/\sinh x$
$y = \coth x = 1/\tanh x$

$y = \sinh^{-1}x = \operatorname{arsinh} x$
$y = \cosh^{-1}x = \operatorname{arcosh} x$
$y = \tanh^{-1}x = \operatorname{artanh} x$
$y = \operatorname{sech}^{-1}x = \operatorname{arsech} x$
$y = \operatorname{cosech}^{-1}x = \operatorname{arcosech} x$
$y = \coth^{-1}x = \operatorname{arcoth} x$



♦ IMPLICIT REARRANGABLE

These are rearranged internally as y = f(x). If the equation contains '±', the graph is treated as two separate functions, a 'positive' and negative' branch.

$2x + 3y = 6$

$2x^2 + 3y^2 = 6$

$y^2 = x^3 + x^2$

$y^2 = x^2(1 - x^2)^3$

$y^2 = x(4 - x)$ and $y = x(4 - x)$

$y^2 = (x - 1)^3/x$

$y^2 = x(x - 2)^2/(4 - x)$

$y^2 = x(4 - x)(x - 2)^2$

$y^2 = x(x^2 - 3) + c$

$y^3 = 1 + x$

$x^3 + y^3 = \pm 1$

$x^3 - y^3 = \pm 1$

♦ CONICS

Note: Autograph uses 'e' in the context 'e^x', so another letter needs to be used in equations for eccentricity.

Circle $(x - a)^2 + (y - b)^2 = r^2$ *Centre:* (a, b)
[e = 0] $x^2 + y^2 + 2gx + 2fy + c = 0$
Centre: $(-g, -f)$ $r^2 = g^2 + f^2 - c$

Ellipse $x^2/a^2 + y^2/b^2 = 1$
[e < 1] $x = a\cos\theta$, $y = b\sin\theta$
Focus: $(\pm ae, 0)$
Directrices: $x = \pm a/e$

Parabola $y^2 = 4ax$ *Focus:* (a, 0)
[e = 1] $x = at^2$, $y = 2at$
Directrix: $x = -a$

Hyperbola $x^2/a^2 - y^2/b^2 = 1$
[e > 1] $x = a\sec\theta$, $y = b\tan\theta$
Focus: $(\pm ae, 0)$
Directrices: $x = \pm a/e$

Rectangular Hyperbola $xy = c^2$ $x = ct$, $y = c/t$
[e = $\sqrt{2}$] $x = ct$, $y = c/t$
Focus: $(\pm\sqrt{2}c, \pm\sqrt{2}c)$
Directrices: $x + y = \pm\sqrt{2}c$
Asymptotes: $y = \pm(b/a)x$

Polar Conic $1/r = 1 + k\cos\theta$
[e = k]

General $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

♦ GENERAL IMPLICIT

These cannot be re-arranged or analysed as y=f(x) and are plotted using a sampling process.

$\sin x + \cos y = k$

$\sin x = \cos y$

$xy^2 - x - y = k$

$x\hat{}y = y\hat{}x$

$x^3 + y^3 = 3xy$

$(x^2 + y^2)^2 = x^2 - y^2$

$(x - y)(x + y) = 0$

$x^2 + 2hxy + y^2 = 9$

$(x^2 + y^2 - 4) + k(2x - 3y + 2) = 0$

$a(x^2/9 + y^2/4 - 1) + b(x + y + 1) = 0$

♦ FUN EQUATIONS

$y = x\sin(1/x)$

$y = \sin x/x$

$y = \text{int}(x)$

$y = |x| \pm \sqrt{(1 - x^2)}$

$x^2 + (y - mx^2)^2 = 1$

$x^4 + x^2y^2 + y^4 = x(x^2 - y^2)$

$(2y - x)^2 = x\cot x$

and $2y - x = 0$

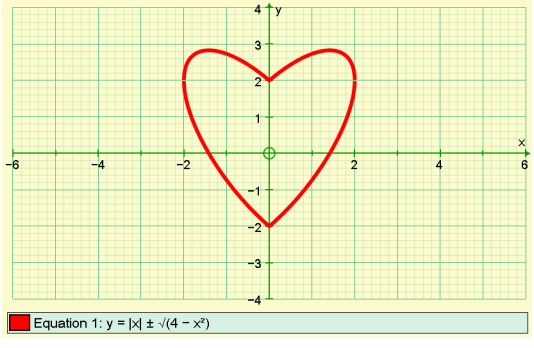
$xy = x^3 \pm 1$

$y = 2(x^2 + |x| - 6)/(3(x^2 + |x| + 2)) \pm \sqrt{(36 - x^2)}$

♦ INEQUALITIES

Use "View" => "Preferences …" to set whether the 'accept' or the 'reject' region is shaded. The default shades the 'reject' region, leaving the 'accept' region clear to mark solution points by adding points.

Straight Lines $y < 3x + 1$ $x \geq -2$
$2x + 3y \geq -3$ $y < x^2$



Other $x^2 + y^2 < 16$

♦ FUNCTION DEFINITIONS

Use the button or the Function Definitions entry in the Equation menu.

f(x) = sinx then y= f(−x)
y = f(|x|)
y = f(f(f(x)))
f(x) = x² with g(x)= (f(x + h) − f(x − h))/(2h)
then
y = g(x) for various 'h'

♦ PARAMETRIC EQUATIONS

Parameter time ('t'):
x = at², y = 2at
x = ut, y = vt − gt²
x = 18t − 1.5t², y = −24t + t²

Trigonometric ('t' or 'θ') [use ALT−T to enter θ]
x = sin2t, y = cost
x = sin(at), y = cos(bt)
y = asin(nθ), x = bcos(nθ)
x = t − sint, y = 1 − cost [Cycloid]
x = t − 2sint, y = 1 − 2cost [Prolate cycloid]
x = 7cost − cos7t, y = 7sint − sin7t [Epicycloid]
x = sint, y = sin(t + 30) [in degrees]
x = 2cot(t), y = 2sin²(t) [Witch of Agnesi]

♦ POLAR EQUATIONS [use ALT T to enter 'θ']
r = 2cos4θ
r = sinθ [circle]
r = secθ [straight line]
r = ±2sin2θ
r = 1, with θ-step = 90° [Square]
r = 3/(2 − cosθ) [Ellipse]
r = 1 − cosθ [Cardiod]
r = e^(θ/4)
1/r = 1 − kcosθ [see Conics]
r² = θ/20 [Fermat's spiral]
r² = cos2θ [Lemiscate of Bernoulli]
r² = lnθ

Polar 'radials' can be entered in the form θ = ...:
θ = nπ/8 and r = cos(8θ) [n = 1 to 8]

♦ PIECEWISE EQUATIONS

Use the equation entry startup options to enter the left hand limits and final for each segment of the equation:
y = −1, cosx,1

Startup options: [−2, −π/2, π/2, 2]

♦ EVOLUTES

Parent		*Evolute*
$y = x^2/4$	[Parabola] =>	$x^2 = (4/27)(y - 3)^3$
$x^2/4 + y^2/1 = 1$	[Ellipse] =>	(2x/3)^(2/3) + (y/3)^(2/3) = 1
$x^2 - y^2 = 1$	[Hyperbola] =>	(x/2)^(2/3) − (y/2)^(2/3) = 1

♦ 1st ORDER DIFFERENTIAL EQUATIONS

Explicit:
y' = −x/y
y' = x+y
y' = 2xy/(x² − y²)
y' = sin(xy)
dy/dx = y

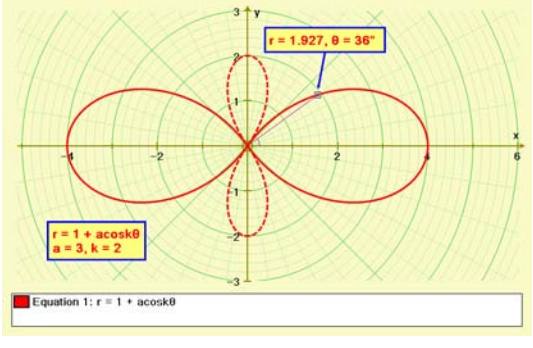
Implicit:
y' + ky = 1
y' + y = x
y' + y = 2sinx

With axes reset to x-t: x' enters ẋ, eg:
ẋ + x = 2sint

♦ 2nd ORDER DIFFERENTIAL EQUATIONS

With axes reset to y-x:
y'' = 1
y'' = y
y'' + 2ky' + y = 0
y'' + 2ky' + y = x

With axes reset to x-t: x' enters ẋ, x'' enters ẍ, e.g.:
ẍ = −10
ẍ + n²x = 0 [SHM]
ẍ + 2λẋ + n²x = 0 [Damped SHM]



♦ 3D: z = f(x, y)

z = asinxcosy [Egg Box]
z = x² + y² [Paraboloid]
z = x² − y² [Saddle]
z = xy [Saddle]

Linear equations that can be rearranged to z = f(x, y):

ax + by + cz = d Plane, ⊥ vector [a, b, c]

♦ 3D: IMPLICIT

Created using a scan of the bounding cube. The resulting surfaces are not available for any calculations, and points cannot be placed on their surface.

Second-degree equations:

x² + y² + z² = r² [Sphere]
x²/a² + y²/b² + z²/c² = 1 [Ellipsoid]
z² + y² = x² [Cone, axis: x−axis]
x² + z² = y² [Cone, axis: y−axis]
x² + y² = z² [Cone, axis: z−axis]
y² + z² = r² [Cylinder, axis: x−axis]
x² + z² = r² [Cylinder, axis: y−axis]
x² + y² = r² [Cylinder, axis: z−axis]
xyz − yz − xz − xy + x + y + z = 1 [Three planes]
x² + y² + a = z² [Hyperboloid]
x² + y² = a²(cosh(z/a))² [Catenoid]
z² + (Ä(x² + y²) − b²)² = a² [Torus]

Other implicit equations:

cosx + cosy + cosz = 0 [Schwarz's P surface]
|x| + |y| = r [Box]
cosxsiny + cosysinz + coszsinx = 0 [Gyroid]
e^zcosx − cosy = 0
|xyz| = a

CONIC SECTIONS:

Note - the Cone can be input as

r = z (cylindrical coordinates).
z = 2 [Circle]
z = 2+x/2 [Ellipse]
z = 2+x [Parabola]
z = 2+2x [Hyperbola]
x = 2 [Rect. Hyperbola]
x = 0 [Two Lines]
z = x [One Line]
z = 0 [Point]

♦ 3D: PARAMETRIC

[ALT T = 'θ']

x = t, y = ±t, z = 0 [pair of straight lines]
x = sinθ, y = cosθ, z = θ [spiral]
x = a + kcosθ, y = θ, z = b + ksinθ

♦ 3D: SPHERICAL POLARS

[ALT T = 'θ', ALT F = 'ϕ']

r = 1 [sphere]
r = 3sin2θ

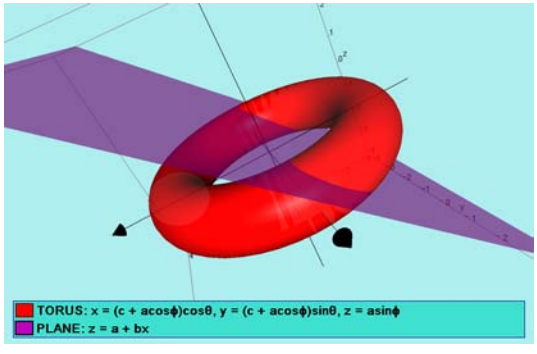
Spherical polar equations may be entered parametrically: x = f(θ,ϕ), y = f(θ,ϕ), z = f(θ,ϕ)

x = a + kcosθ, y = ϕ, z = b + ksinθ
[offset cylinder, radius 'k']
x = (c + acosϕ)cosθ, y = (c + acosϕ)sinθ, z = asinϕ
[Torus: radius 'c', sub−radius 'a']
x = (2 + 0.2sinπθ)sinπϕ, y = 0.2cos2πθ + 3cos2πϕ,
z = (2 + 0.2sin2πθ)cosnπϕ [Lissajous]
x = ksin(θ)sin(ϕ), y = kcos(θ), z = ksin(θ)cos(ϕ)
[Sphere]

3D: CYLINDRICAL POLARS

[ATL T = 'θ']

r = 1 [cylinder]
r = 1 [triangular prism, θ−step set to 2π/3]
r = 1 [triangular box, θ−step set to π/2]
r = z [cone]
r = (1−0.25z²)(1 + 0.5sin(1.5πz) + 0.3cos5θ)



Adjusting for the Visually Impaired

1. Using Windows accessibility facilities:
Start => Programs => Accessories => Accessibility.
Use the Accessibility Wizard, or the Magnifier or Narrator options.

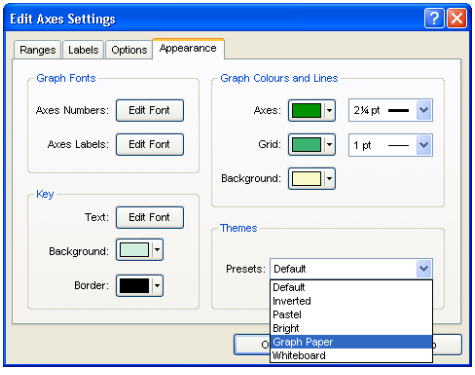
2. Make Autograph Toolbar icons bigger:
Right-click over the toolbars
=> “Customize” => “Options” => tick “Large Icons”.

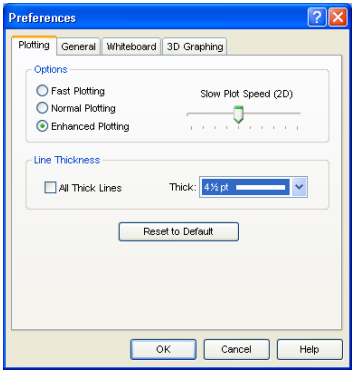
3. Make the Autograph axes more visible:
Use “Axes” => “Edit Axes” => “Appearance”.
Here you can make the axes numbers, the axes labels and the equation list much easier to read by setting the font size, say, to 18pt and bold. Also the axes and grid lines can be thickened up.

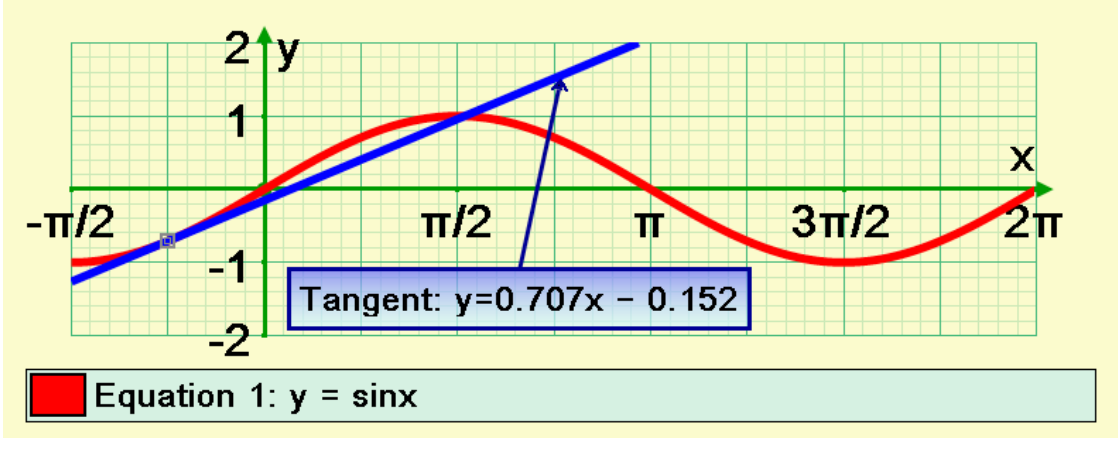
It is suggested that users save three files “Blank-1”, “Blank-2” and “Blank-3” for each of the 1D, 2D and 3D pages, each set up for ideal visibility. Load the appropriate file at the start of each new session.

4. Make the Autograph plotting thicker:
Whereas Axes Appearance is set for each page (hence the suggestion to save ‘blank’ pages), it is possible to set the general line thickness for all pages created by the current user.

Use “View” => “Preferences” => “Plotting” and tick “All Thick Lines”. A setting of 4½pt will make all plotted lines and curves more visible.







Autograph 3.20 on an Apple Mac

Power PC Based Mac

Autograph 3.20 will install and run on an Power PC Based Mac by making use of the ***Virtual PC*** application, published by Microsoft. This is a Windows Emulation program. It lets your Mac run Windows. If you already have a licensed installer for Windows then you can just buy ***Virtual PC*** on its own. If you don't then you can buy ***Virtual PC*** bundled with ***Windows XP***.

It's simple to drag and drop files between the two environments. So you can capture ***Autograph*** screens and incorporate them in Mac documents. The 3D facilities do not work with the current version of Virtual PC and Microsoft have stated they will not be bringing out any future versions.

Intel Mac

Parallels Desktop allows you to run Windows on Mac OS X without rebooting. Codeweavers produce CrossOver which allows you to install Windows applications directly on your Mac. However, 3D facilities are not expected to work using either of these methods.

Apple's Boot Camp software allows you to run Windows on your Mac but you must restart the computer to switch operating systems. It is expected that 3D facilities will work using this method.

Citrix and MS Terminal Server

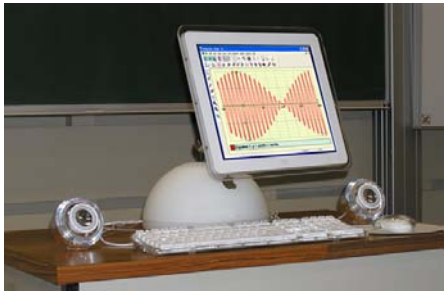
This is a network solution, running Autograph on a PC Server, and using Macs as thin clients. It works, but there may be concerns about speed and, again, difficulty with the implementation of the 3D facilities.

Using the Apple Mouse

The Standard Apple mouse has only one button, which is the equivalent to the PC LEFT CLICK button. Also, there is also no roller on the mouse.

It is possible to plug in a standard PC mouse to a Mac with OS X, but for a single-button mouse, you can use this conversion table when reading this manual:

PC Mouse/Keys	**MAC Mouse/Keys**
RIGHT-CLICK	CTRL-CLICK or ALT-CLICK or use the 'OBJECT' Menu
CTRL-CLICK	APPLE-CLICK
SHIFT-CLICK	SHIFT-CLICK
CTRL-any key	APPLE-any key e.g. Apple-C = COPY, etc
ALT-any key	CTRL-any key e.g. Ctrl-. = Point mode



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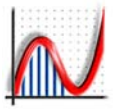

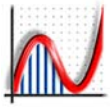

Autograph Version History and Credits

Autograph 3 [PC]

♦ Published in June 2004, V. 3.10: April 2005
V. 3.20 International (Unicode) edition: January 2007

♦ **C++ programming and overall program design:**
MARK HATSELL

♦ **Concept and mathematical specification:**
DOUGLAS BUTLER

♦ **Additional material by**
Mike Pinna, Jamie Collin, Stephen Whipp, Sam Butler and Simon Woodhead

♦ **Testing and training materials by**
Alan Catley, Fred Dye, Alastair George, Roger Harding, Martin Withington, *Overseas:* Jim Claffey (Australia), Kate Rozsa (USA), Mike Wakeford (Norway) and many others.

♦ **Autograph Extras and CD Interface:**
Stephen Whipp

♦ **Autograph Help Files:**
Jamie Collin and Simon Woodhead

♦ **Arial for Autograph Font:**
Ian Bezer, Monotype Imaging Ltd (Redhill)
www.monotypeimaging.com

♦ **Graphic Design:**
Simon Dolby, The Design Factory (Oundle)
www.dolbygallery.com

♦ **Program Icons:**
Dave Wilkinson, Glyph Lab (Cornwall)
www.glyphlab.com

♦ **Autograph Web Site:**
Scott Wright, Dreamshock (Shoreham)
www.dreamshock.com

♦ **TSM Resources web site:**
(Technology for Secondary Mathematics)
from the iCT Training Centre, Oundle School
www.tsm-resources.com

Autograph 2 [PC]

♦ Published in October 2000,
V. 2.10 January 2003

♦ **C++ programming + overall program design:**
MARK HATSELL

♦ **Concept and mathematical specification:**
DOUGLAS BUTLER

♦ **Autograph Extras:**
Mohan Ganesalingham

Autograph 1 [Acorn]

♦ Published in September 1993,
with regular updates until September 1998.

Autograph was originally written in BBC BASIC at Oundle School, Peterborough (UK), based on an original concept by PHILIP COUZENS.

Programming was under the direction of Douglas Butler, with major contributions from Adrian Peakman and Alex Stanhope.

Support Materials

♦ Autograph and support materials:
EASTMOND PUBLISHING Ltd. (UK)

Web: **www.autograph-maths.com**
and **www.autograph-math.com**
Email: **support@autograph-maths.com**

♦ International marketing and teacher support materials:
CHARTWELL-YORKE Ltd. (UK)

Web: **www.chartwellyorke.com**
Email: **sales@chartwellyorke.com**
Contact: Philip Yorke