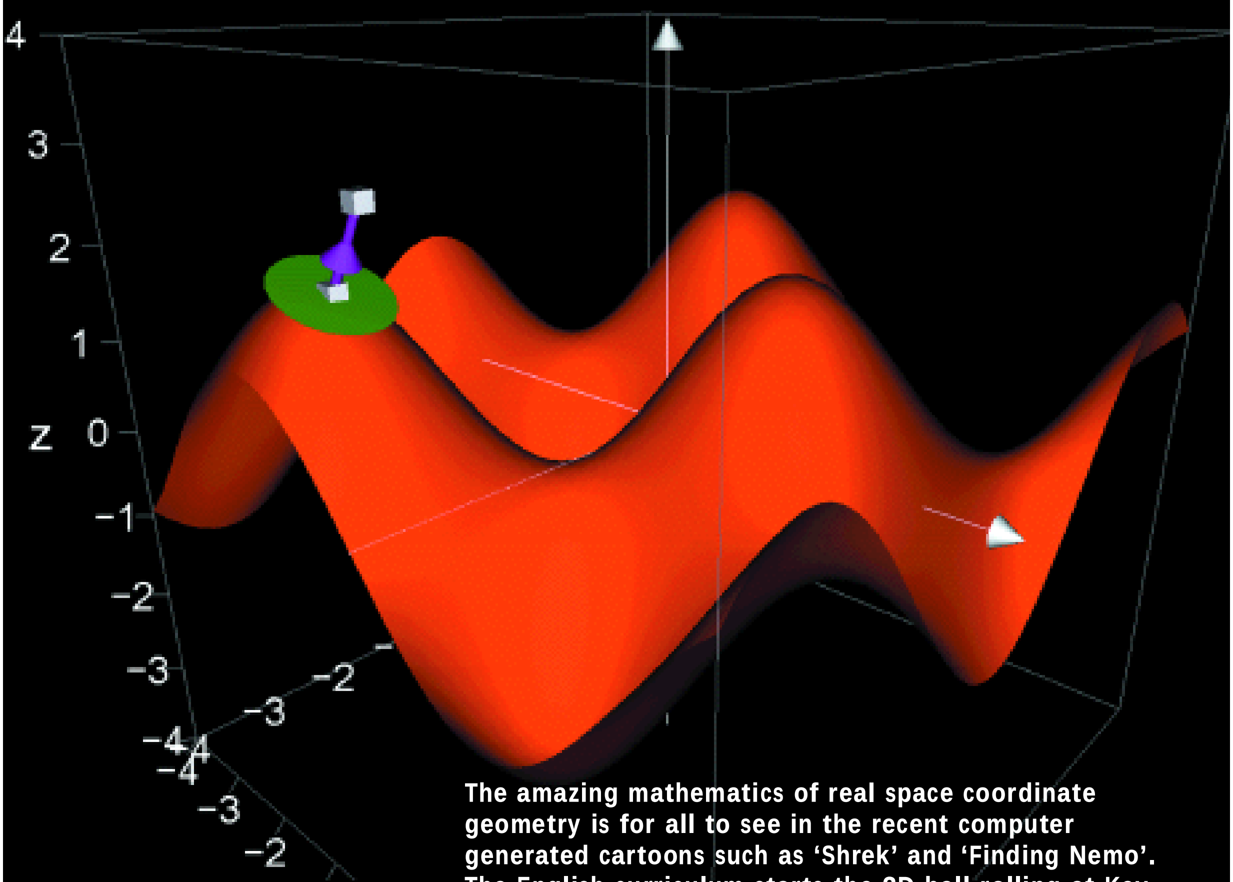
Time to think 3D

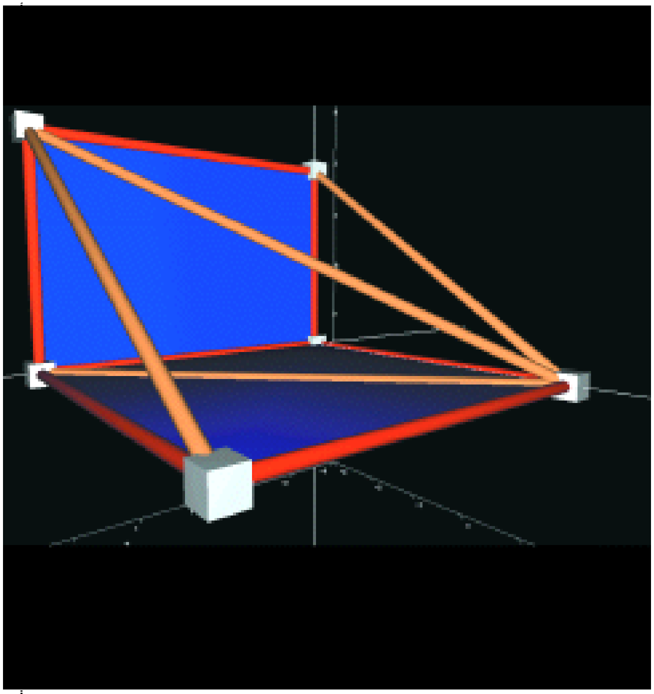
Douglas Butler

How Autograph has been extended to help with visualisation of 3D concepts.



The amazing mathematics of real space coordinate geometry is for all to see in the recent computer generated cartoons such as 'Shrek' and 'Finding Nemo'. The English curriculum starts the 3D ball rolling at Key Stage 1 with "Observe common 3D shapes ...", with KS2 extending to prisms and pyramids.

Key Stage 3 introduces the idea of 3D coordinates, whereas KS4 (Higher) moves on to using trigonometry and Pythagoras in 3D, and the angle between a line and a plane.



Post 16, the top Pure paper develops the use of vectors in 3D, and vector equations of lines and planes. Further Mathematics students look at cross product, intersection of planes, skew lines, shortest distances, and surfaces. 3D work is also common in many other countries at school level, including the USA (AP Calculus) and the IB.

So how do teachers get these ideas across? Excluding the distant prospect of classroom holography, the viewing surface will always be two dimensional. Mark Hatsell, the programmer behind *Autograph*, has long had the idea that the software of the games industry could help with 3D visualisation in the mathematics classroom. Direct-X, the force that drives popular car racing and military simulations, offers the chance to create a smooth and realistic 3D scenario for studying coordinate geometry and surfaces.

The first decision in designing an object-based 3D component in *Autograph* was to have a bounding cube, so that users could rotate the scene and look round and behind the objects. Next was to extend the principles of dynamic object dependency already established in the 2D version of *Autograph*, so that 3D objects could easily be created and selected.

So we can put on two points: create a vector or line; put on three points: create a plane; enter an equation: create a surface. We are away!

Vector Equation of a straight line

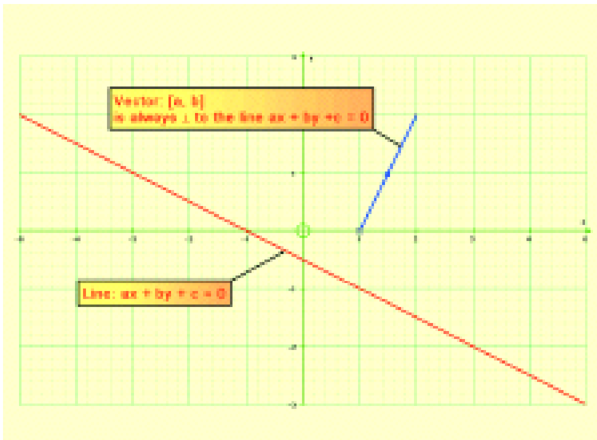
I have always tried to use the same approach to vectors in 2D as in 3D, to make the transition as painless as possible! So a line in 2D or 3D can be visualised as a locus of points formed from a point and a scalar multiple of a vector:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \end{pmatrix} \quad \text{or} \quad \frac{x-a}{l} = \frac{y-b}{m}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} l \\ m \\ n \end{pmatrix} \quad \text{or} \quad \frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$$

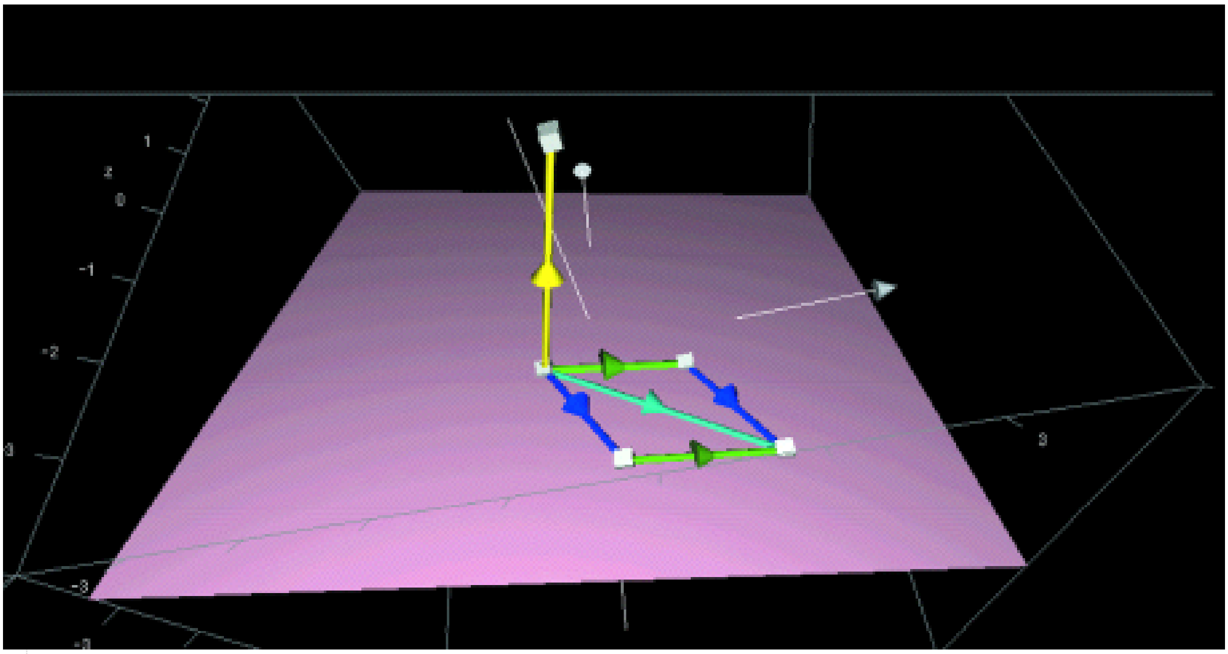
So in *Autograph*, you can create a line in 2D or 3D either by selecting a point and a vector, or by entering in the vector form above.

Vector Equation of a plane



The implicit form of a straight line in 2D is first met by younger pupils in the solution of simultaneous equations. When a line is written in the form ax + by = c it can be shown that the vector [a, b] is always perpendicular to the line, whatever the values of ‘a’ and ‘b’. The equivalent result in 3D is that the vector [a, b, c] is always perpendicular to the plane ax + by + cz = d.

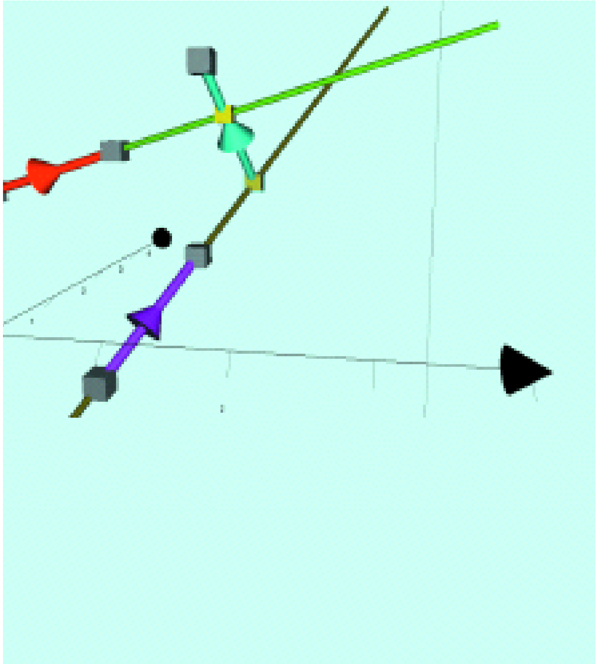
Further Mathematics students get the chance to explore the vector equation of a plane, which turns out to be a natural development of the fact that the linear combination of any two vectors is always in the same plane - something which again



can be demonstrated dynamically in 2D first, where the plane is that formed by the 'x' and 'y' axes. In 3D it can further be demonstrated that the cross product of 2 vectors in the plane is perpendicular to the plane.

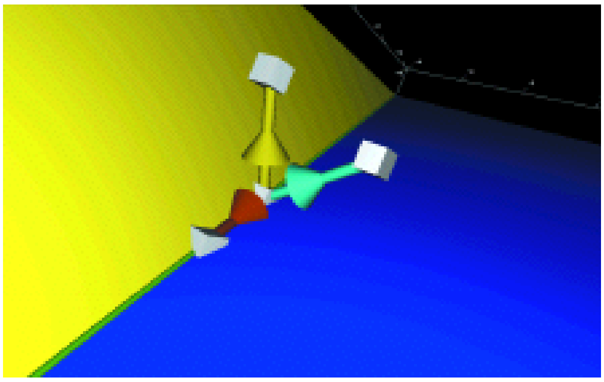
Two Skew Lines

Teachers should never feel to need to abandon theatrical gestures such as waving rulers in the air, but they can now support their efforts by constructing 2 skew lines and the shortest distance using *Autograph*. They can show that the cross product of 2 vectors, one in each line, is coincident with the shortest distance, which is mutually perpendicular to both lines.



Intersection of two planes

The cross product is also a useful tool in illustrating that the line of intersection of two planes is mutually perpendicular to the two normal unit vectors.



And finally surfaces:

That just leaves the world of surfaces: looking at conic sections, turning points and saddle points, cylindrical and spherical coordinates, etc. All bread and butter of course to the software engineers who created Nemo!

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All images in this article have been recorded as flash demonstrations on http://www.autograph-maths.com/micromath.html