# INTERMEDIATE AUTOGRAPH

**INTERMEDIATE AUTOGRAPH WEBINAR,** delivered in four 1½-hour session in July 2020 by Douglas Butler (iCT Training Centre, Oundle) and Rob Smith (La Salle Education)




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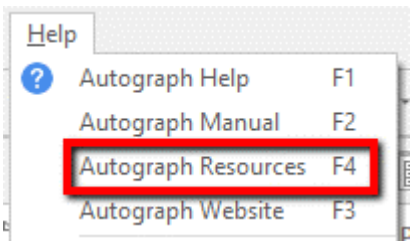
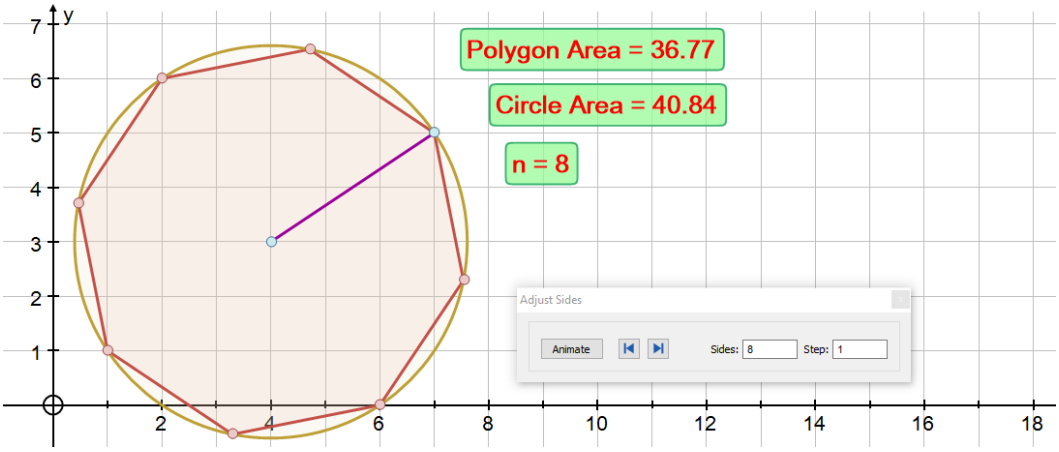



Autograph Resources: Press F4 ->
www.tsm-resources.com
Complete Mathematics Webinar Documentation and .agg files:
https://completemaths.com/autograph/webinar-materials

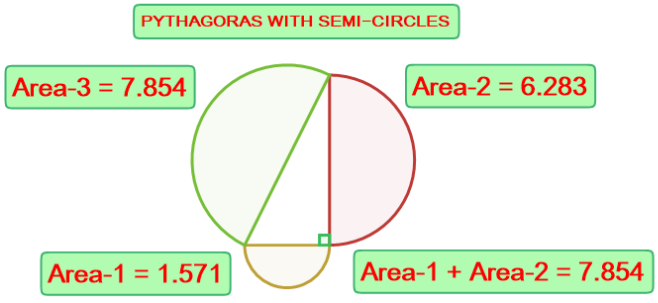
# SESSION 5: Circles and Polygons

## 1. Limit of n-sided polygon




Axes: equal aspect, don't show key
Circle: select CENTRE and POINT, radius is determined by the 2 points.
Polygon: select POINT and CENTRE (that will get fixed!), set 'n' = 5
Calculator: Polygon area =, Circle Area = and 'n' =
Select the polygon and the ANIMATION CONTROLLER. Increase 'n'
**Autograph file: 1. Polygon-circle.agg**

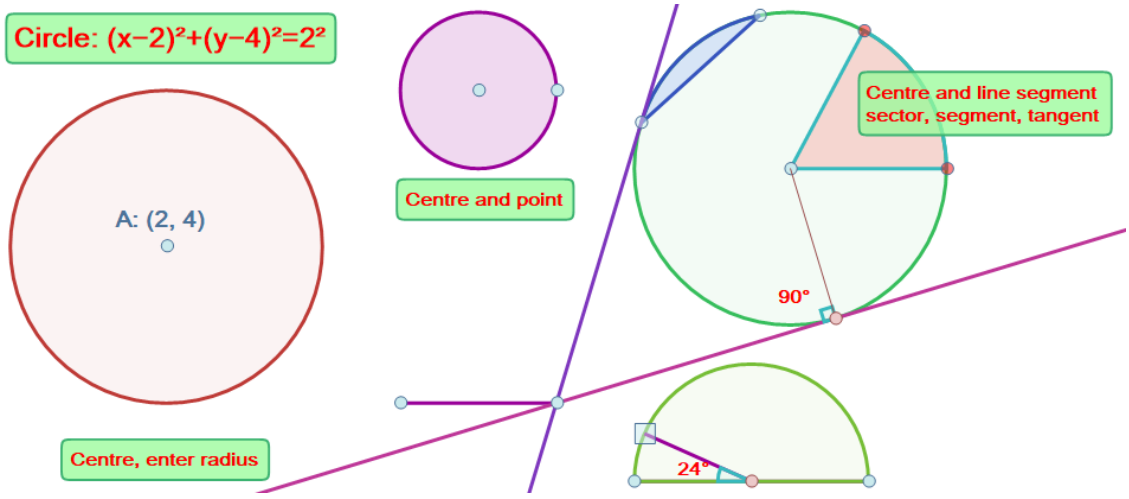
## 2. Pythagoras' Theorem with semi-circles




Axes: equal aspect, no axes, don't show key, and snap settings 1, 1
Add any 3 points to form right-angled triangle
Semi-circles on each side. Calculator results as shown
Repeat with a SQUARE on each side

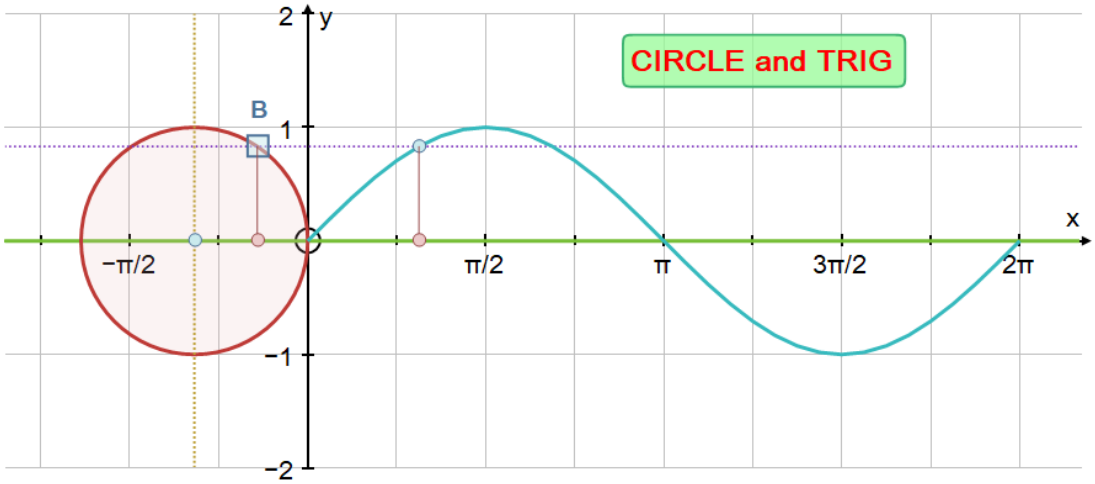
**Autograph file: 2. Pythagoras with semi-circles.agg**

## 3. Various ways to create circular objects in Autograph




**Autograph file: 3. Circles various.agg**

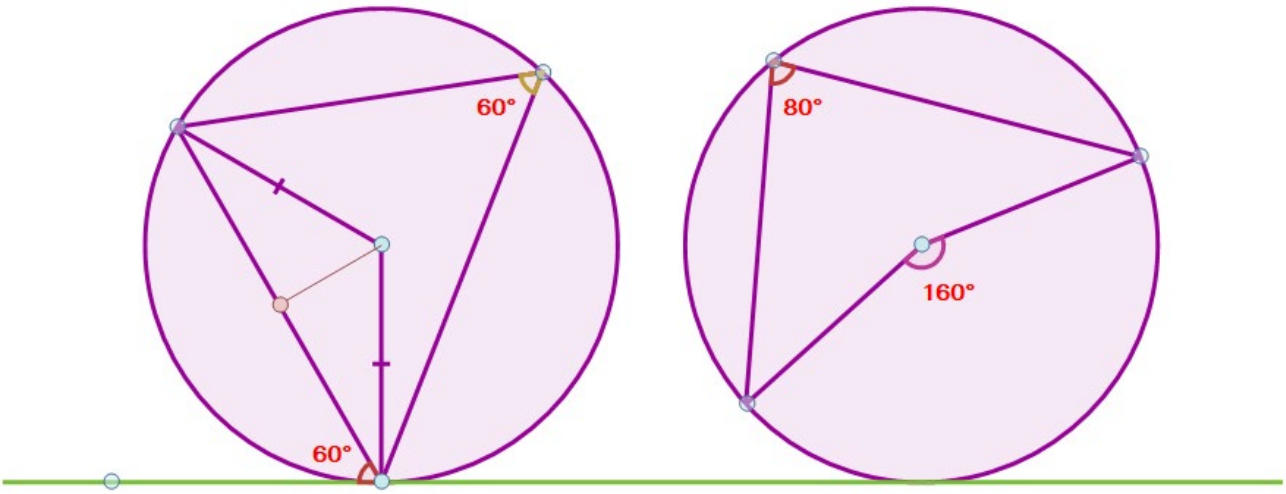
## 4. The unit circle and trigonometry




Draw circle, centre (-1,0), radius 1. Axes: set to π scales, equal aspect
Draw the x-axis by entering equation y = 0
Point B on the circle

Allocate constants: a to the angle of B (B:t), and b to the 'y' value of B.
Draw point (a, b). Select it and B and create LOCUS.
Select 'B' and x-axis: right-click point option "Closest Point". Repeat for (a, b)
Drag 'B' round the circle.

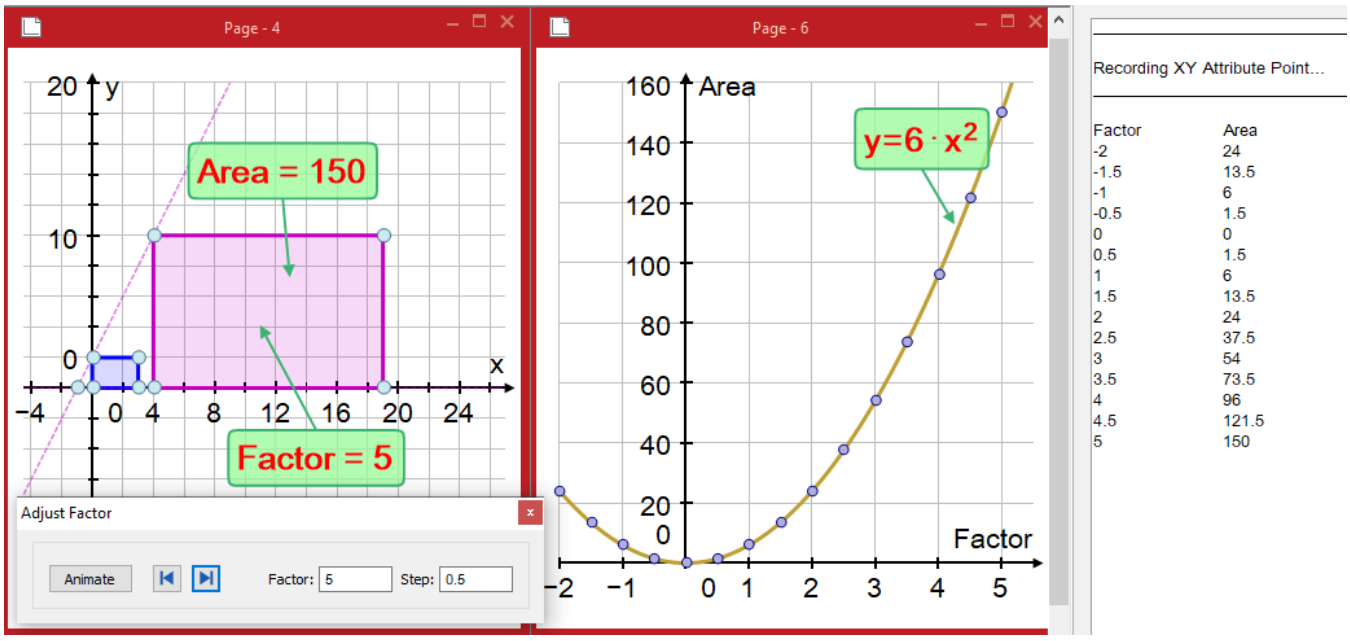
## 5. Circle theorems various




Equal aspect, no axes
Use extended point tool to create circles and line segments
Remember the rule for measuring angles:
select 3 points in such a way that the angle is created in a clockwise manner.

# SESSION 6: Things Parabolic

## 1. Factor and enlargement




Create a shape, either from a set of points or one of the built-in shapes
In the diagram above, the rectangle shape used is 2x3 => area of 6.
Ener a point (-1,0), select the point and the shape: enlargement, scale factor 2
Satisfy yourself that the enlarged area is 4 x the original area, 4 being the factor squared.

Select the enlarged area and use the ANIMATION CONTROLLER to wind back the factor to –2nd and set the STEP to 0.5

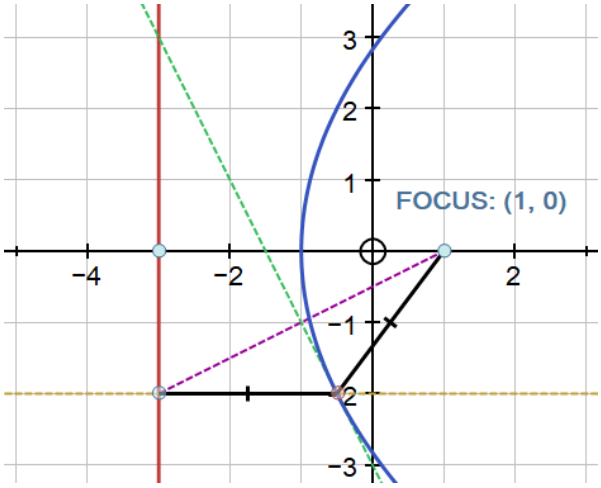
XY Select the enlarged area and XY attribute point:
Set 'x' to be the factor, and 'y' to be the enlarged area. Tick RECORD.
Animate the factor from –2 to 5 and observe the RESULTS BOX (at the right side)

Select the data – new 2D page – enter XY data. Plot
Note it is parabolic, and should fit the formula A = Area1*factor², ie y = 6x²

NOTE: to have the two pages showing side by side, proceed as follows:
Open the page you want on the LEFT, then go to Window => Windows... => select the two you want (using Ctrl if not adjacent), the "Tile Vertically" then OK.

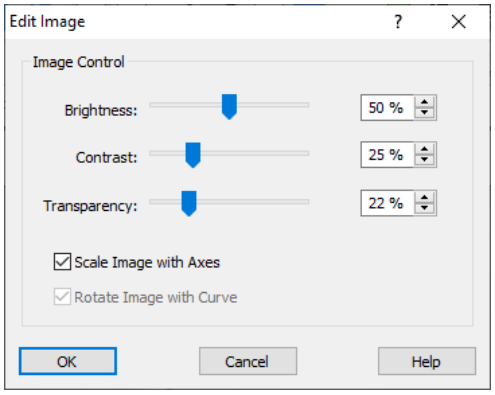
**Autograph files:** **1a. Enlargement.agg** and **1b. Enlargement recording.agg**

## 2. Parabola Construction




**Autograph file: 2. Parabola** construction.agg

## 3. Rainbow: circular or parabolic?




Paste in or drag in the image

DC on the image to edit:
Set transparency to 20% or so to reveal axes.

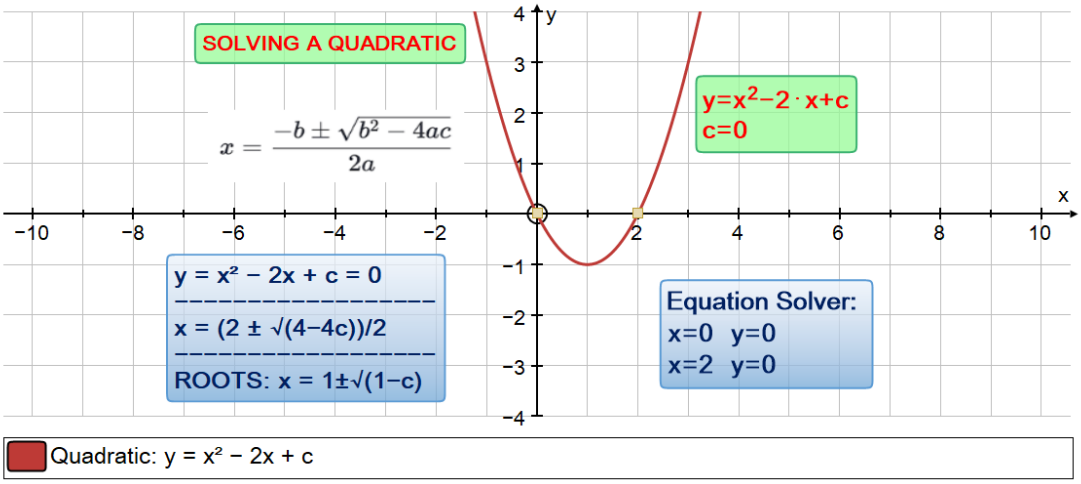
UNTICK "Scale Image with Axes"
in case axes are altered
Remember to TICK it back when saving the file

Place three well-spaced points on the rainbow.
Select them all, then: Create -> Quadratic (3 points), then Circle -> Circle (3 points)
Conclusion? Search the web for "Circular Rainbow"

**Autograph file: 3. Rainbow.agg**

# 4. Solving a quadratic, including complex

## a. 2D page

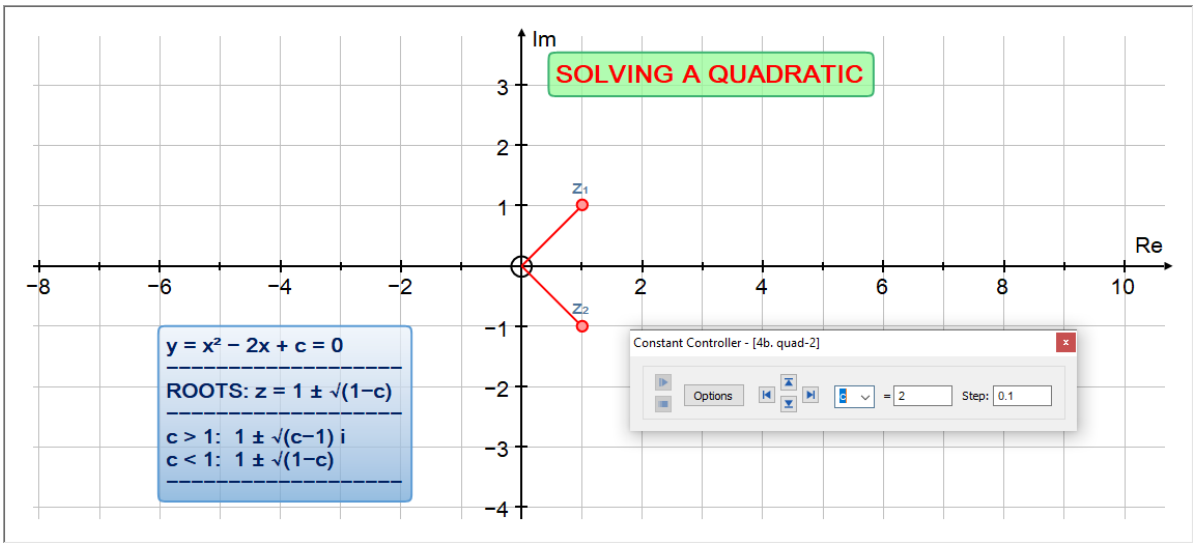



Enter y = x² – 2x + c (note with an even coefficient of 'x' all the twos cancel)
Select the curve, then right-click POINT => Solve f(x) = 0.

Select the roots and the TEXT BOX to show the "Equation Solver"
Roots are x = 1 ± √(1 – c) [note use of Autograph keyboard here], so 'imaginary' when c > 1

**Autograph file: 4a. quad-1.agg**

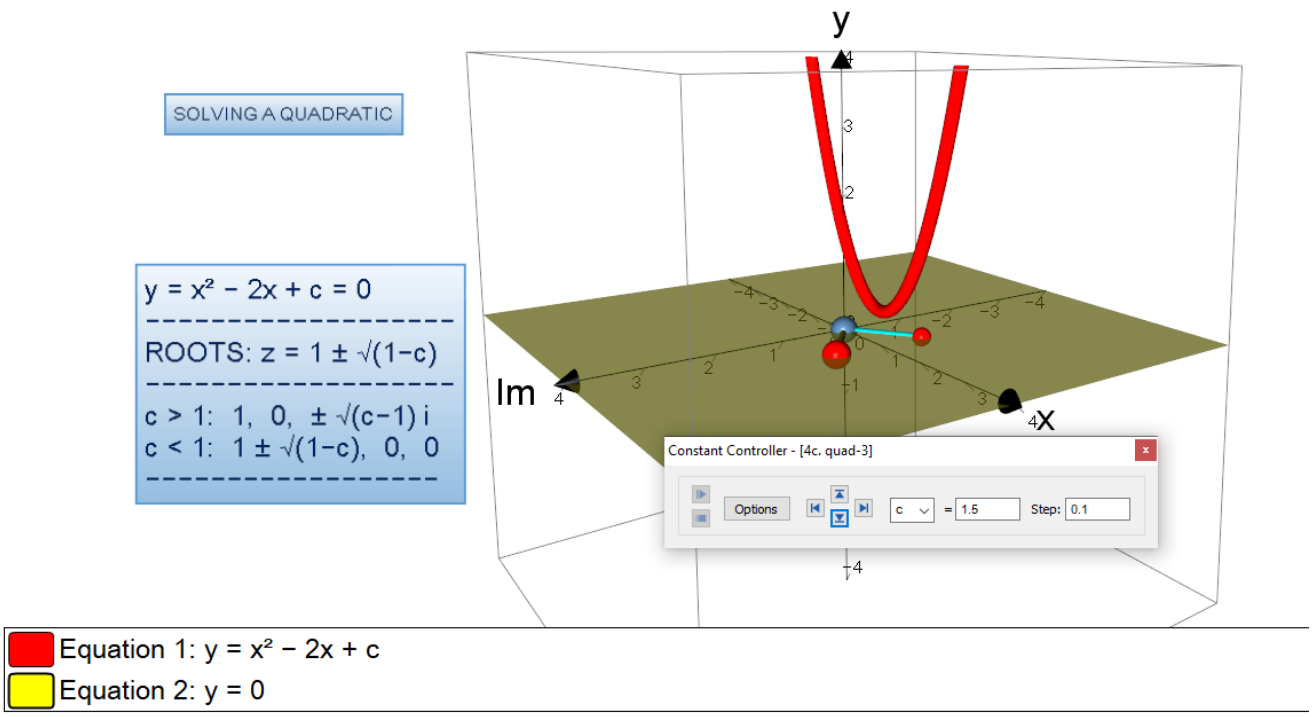
## b. Argand Diagram page




On an Argand Diagram page, enter complex number: z = 1 ±√(1 – c)
'c' will start off = 1. Vary 'c' with the constant controller. Observe!

**Autograph file: 4b. quad-2.agg**

**c. 3D page**




First enter equation: z = 0 (a plane) and change its colour to a lighter colour
Then enter **y = x² – 2x + c,** be sure to tick "Plot as 2D equation"

Edit Axes: rename the '**z**' label as "**Im**"

(,) To show the solutions you need to enter four points (y = 0 for all of them):

1. Real solutions for c < 1: (1+√(1-c), 0, 0)
   and (1 –√(1–c), 0, 0)

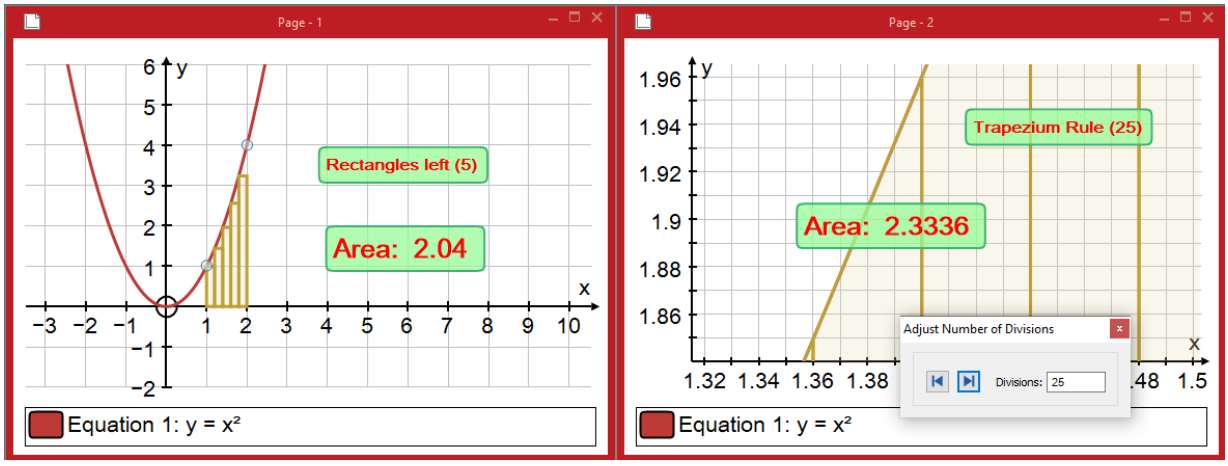
2. Complex solutions for c > 1: (1, 0, +√(c–1))
   and (1, 0, –√(c–1))

Vary 'c' with the constant controller

**Autograph file: 4c. quad-3.agg**

# SESSION 7: Things Numerical

## 1a. Numerical methods: - Trapezium Rule

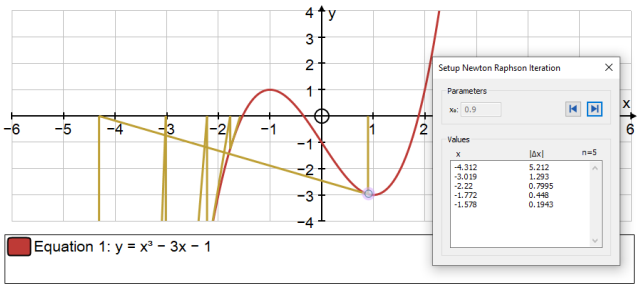



Draw y = x², place points at x = 1 and 2, and Create -> Area with 5 divisions, using Rectangles (left)
Select the area, and Text Box to display "Area = 2.04"
Zoom in a few times; select the area and the animate tool to increase to 25 divisions.
Notice local straightness.
(The answer: 2.333333)

**Autograph: 1b.trapezium rule.agg** and **1c.trapezium rule.agg**

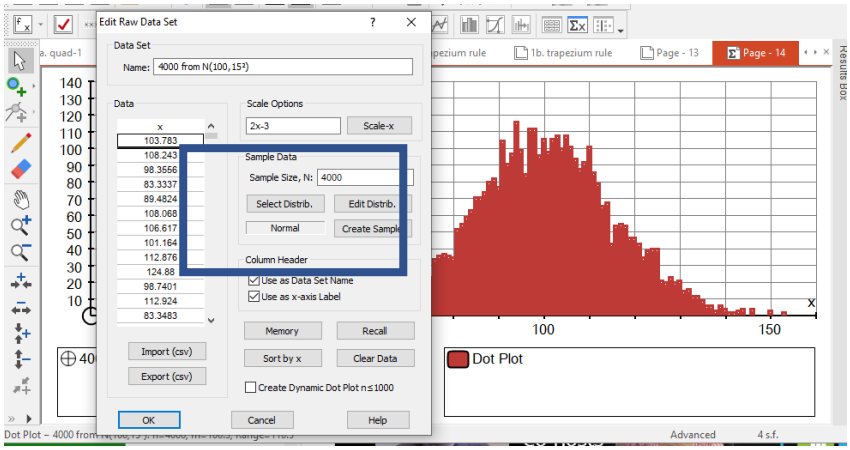
## 1b. Newton-Raphson: watching it fail

Draw the curve y = x³ - 3x – 1,
and place and select a point on the graph
Create -> Newton-Raphson
In the dialogue box, increase the number of iterations.
You can move the point along the curve any time.




**Autograph: 1e. newton-raphson.agg**

## 2a. Creating data

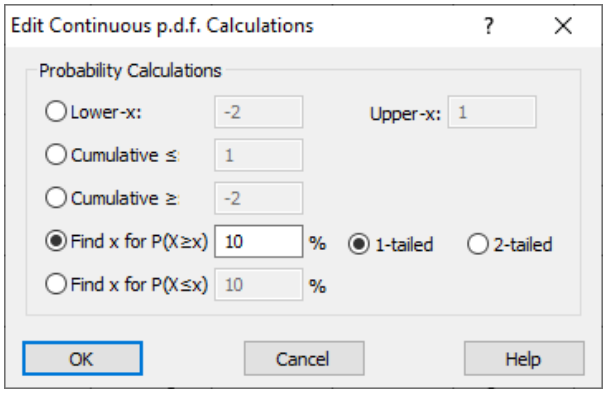


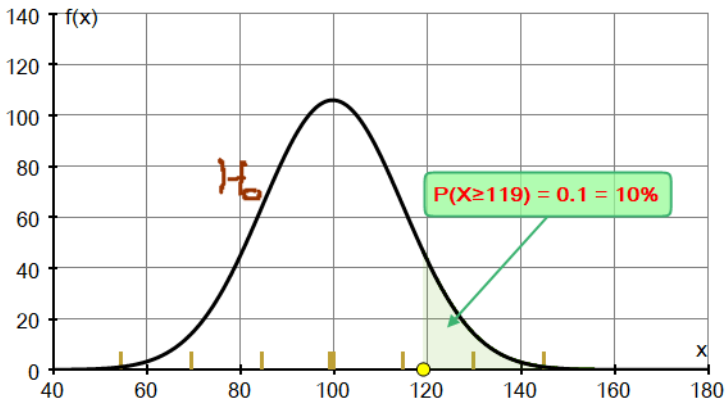

On a Statistics page, right-click "Enter Raw Data". In the "Sample Data" section:
Name: 4000 samples from N(100,15²): Sample size => 4000 Select Distr. => Normal
Edit Distrib. => μ = 100, σ = 15 Create Sample.

Use "Sort by x" to discuss data outside m ± 3 sd, that is <55 and >145
Use "Scale-x" to convert this data into integers [ replace the default "2x–3" with "int(x+½)" ]
OK => right-click: "Dot Plot"

## 2b. Hypothesis Testing

With the above data still in place, right-click: "Enter Probability Distribution"
=> Normal => Edit Normal: "Fit to Data"
Select the Dot Plot -> Hide Object, leaving the Normal plot.
The "Mean ± 3 SDs" option is useful. Select the normal plot "Probability Calculations"

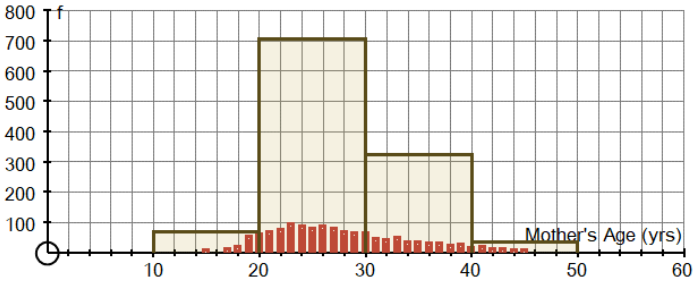



Explore data and hypothesis testing for other distributions:
Discrete: Rectangular, Binomial, Poisson, Geometric, User [will check that Σp = 1]
Continuous: Rectangular, Normal, f(x) [enter your own, it ensures that the total area = 1]

**Autograph: 2a. creating data.agg and 2b. hyp-test.agg**

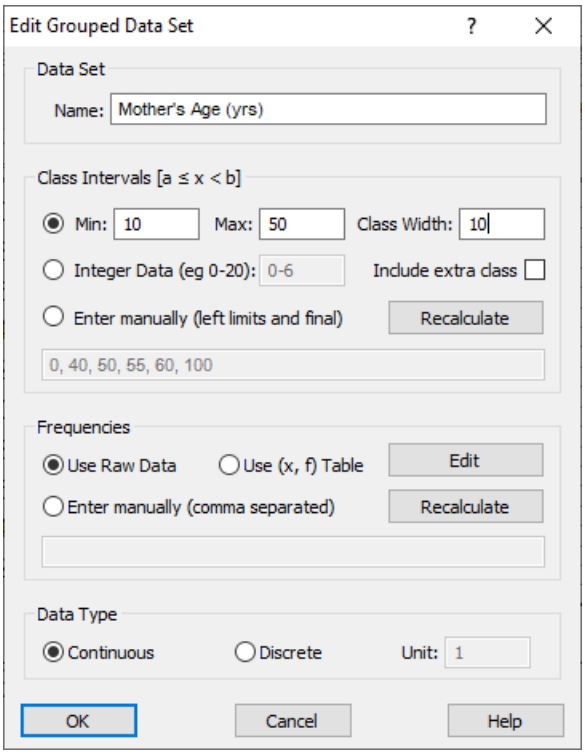
## 3a. Importing data from Excel: Baby Data



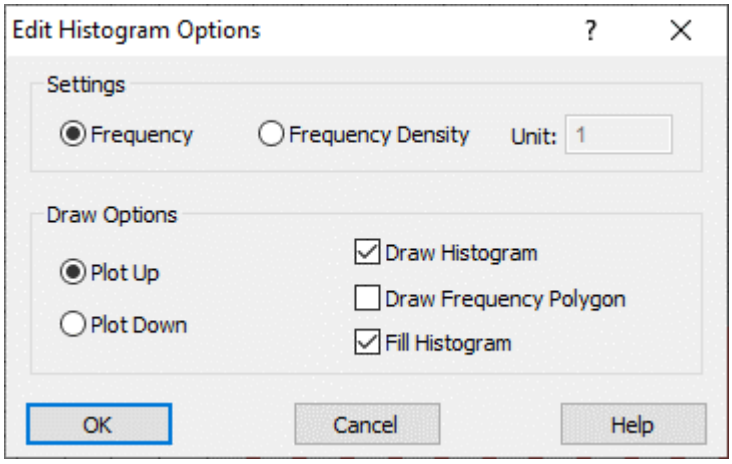

**Excel: 3a. BabyWeightData(1132).xlsx**
**Autograph: 3a and 3b.mothers.agg**

In Excel: copy the column "Mother's Age".
in Autograph: paste in "Enter Raw Data".
Plot a "Dot Plot"

In order to plot Histogram, you need first to "Group Data Set"
Carefully consider all the options in this dialogue

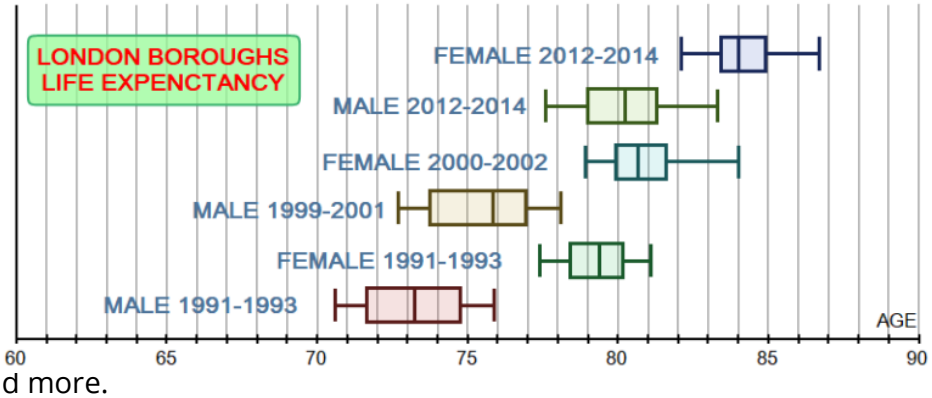



OK -> Histogram
Carefully consider these options too:




## 3b. Importing data from Excel: Multiple Box Plots

**Excel: 3b. boroughs-life.xlsx**
**Autograph:**
**3b. multiple box plots.agg**




Excel:
select one data for life expectancy.
Autograph: Enter Raw Data - OK
then Box and Whisker Diagram

This can be manipulated as you add more.

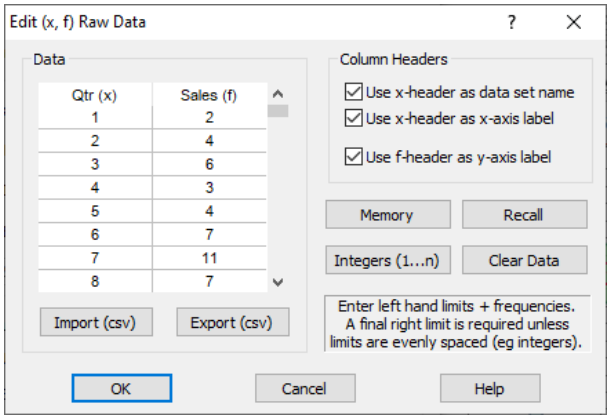
## 4. Something new: Moving average

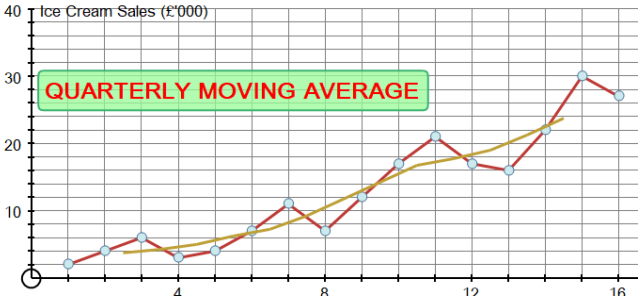
**a. Ice cream sales**
**Autograph: 4a. IceCreamSales.agg**
**Excel: 4a. IceCreamSales.xlsx**

Excel: copy the x-f columns
Autograph: On a Statistics page choose
"Enter Grouped Data" (because it is frequency data)
Choose "Use x-f Table" => Edit
Paste the data and tick the 3 boxes. OK

Choose discrete Data (important for Line Graph)
OK. Right-click: Line Graph

Select the Line Graph -> Moving Average (Unit=4)



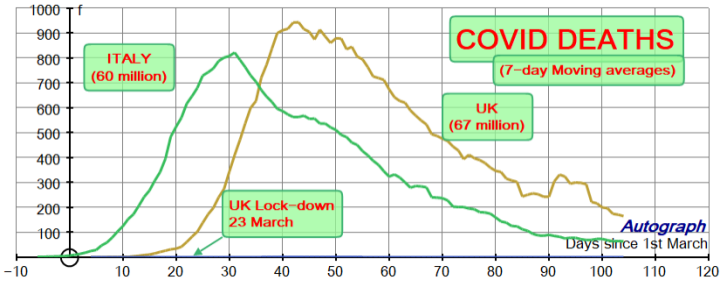

**b. Covid statistics**
**Excel file: 4b. covid-UK-Italy**
**Autograph file: 4b. covid-UK-Italy.agg**

Excel: UK data - Copy Columns 'B' and 'C'
As above:
Autograph: "Enter Grouped Data": use x-f
Select "Discrete Data"
OK – plot and select "Line graph"

"Moving Average"
– unit set to 7 (weekly data)

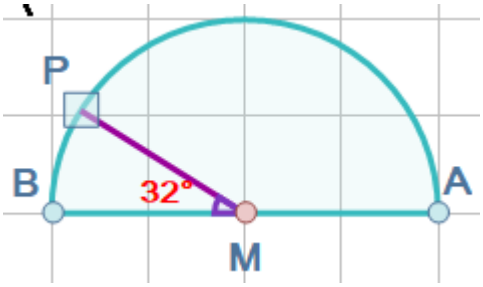
Repeat for Italy. Consider scaling the Italy results to match the UK population.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Date | Since 1 Mar | UK Deaths | |
| 2 | 01-Mar-20 | 1 | 0 | |
| 3 | 02-Mar-20 | 2 | 0 | |
| 4 | 03-Mar-20 | 3 | 0 | |
| 5 | 04-Mar-20 | 4 | 0 | |
| 6 | 05-Mar-20 | 5 | 0 | |
| 7 | 06-Mar-20 | 6 | 0 | |
| 8 | 07-Mar-20 | 7 | 1 | |



# SESSION 8a: TOOLS for Problem Solving

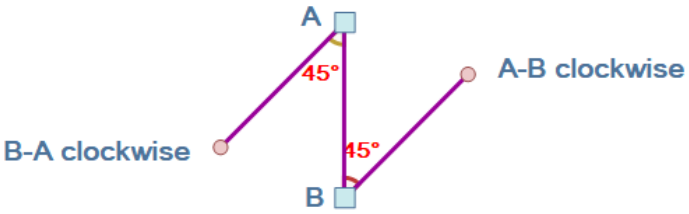
## 1. Creating an angle controller




Select 'A' then 'B' -> Circle -> Semi-circle
-> Mid-point 'M'
Point 'P' on the perimeter, draw radius
Select 'P' – Edit draw options to make it bigger
Select P-M-B -> create angle

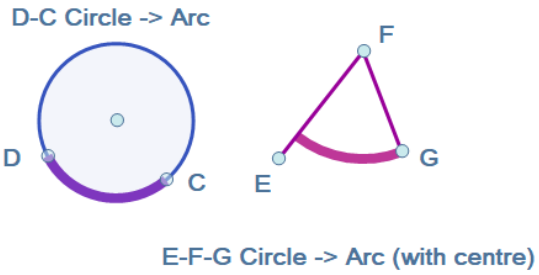
 Manage Constants: associate 'ϕ' with the angle attribute.
Add a new point (ϕ/10,1) and watch it move as you alter the position of 'P'

## 2. Creating an angle from 2 points




Select A then B then clockwise
or
Select B then A then clockwise

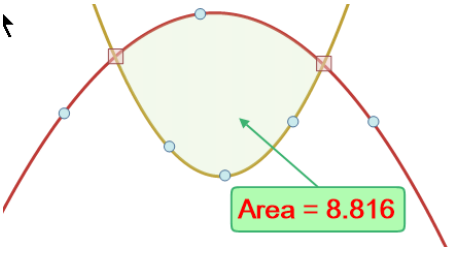
## 3. Two ways to create an Arc Length on a circle




Circle -> Arc [The circle already exists]
Select two points on the circumference in such a way as to create a clockwise object.

Circle -> Arc (with centre) [The circle does not exist]
The first point can be anywhere, the second point Selected is the centre of the circle, and the third Point determines the radius

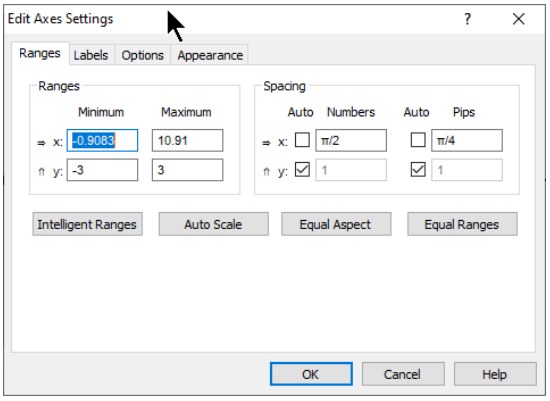
## 4. Area between two created parabolas

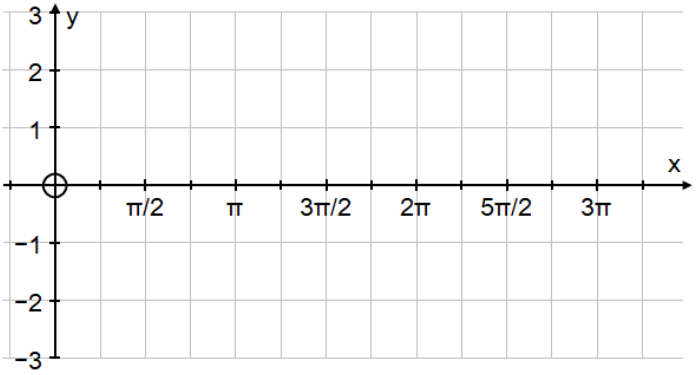



Plot 6 points, select 3 at a time and use "Create -> Quadratic (3pts)" to create two quadratics. Use the intersection tool to find the two intersection points.

Select the two intersection points, then the top curve then the bottom one, to create the area.

## 5. Axes to PI-scales




In 'Edit Axes', set your y-scales first, then 'Equal Aspect'. Remember the x-scales are determined automatically under 'Equal Aspect', depending on the window size.

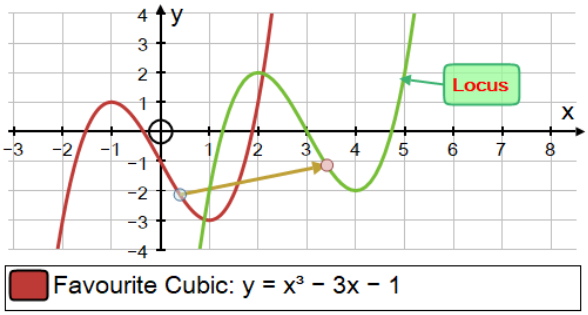
In 'Edit Axes', set 'Numbers' to π/2 and 'Pips' to π/4 *[I am using the Autograph keyboard here!]*

With trig functions plotted, the RED TICK will put π-scales in for you.

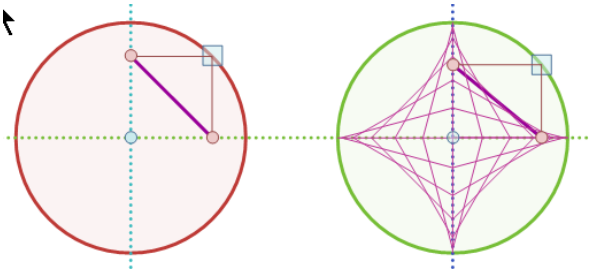
## 5. How to create a Locus

The principle is to select one point that is constrained to move in a path (here along a cubic), and a second point that is related to the first point (here by a vector)

Select both points (in either order) and Choose "Create -> Locus"




You can also create a locus of an object. Here a point moving round a circle is related to a line segment (which itself is a good illustration of $\sin^2 x + \cos^2 x = 1$).

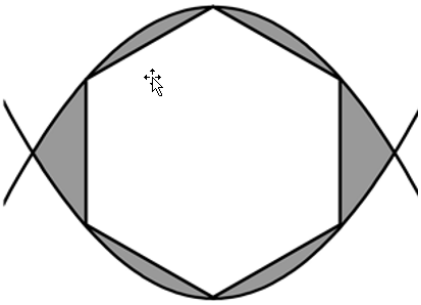


Select the point and the line segment to create the locus. The locus dialogue allows you to alter the step: here π/12 works well.

**See Autograph file: 5a. circle-locus.agg**

# SESSION 8b: Problem Solving

## 1. mathsConf23 problem

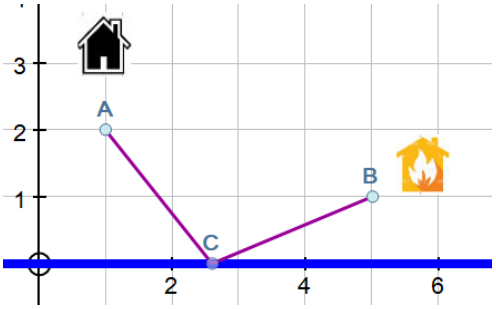


Find the area between a regular hexagon of side 1, and two parabolas, drawn as indicated

From 2 points, vertically spaced by 1, create the hexagon then the two parabolas, then the two intersection points.
Find and display the area between the two parabolas.
Use the calculator to display the hexagon area and the difference.

**Autograph files:**
**1. Area puzzle - start.agg** and **1. Area puzzle.agg**

## 2. Heron's Problem




Find the shortest path from A to B, via the river.
Use the calculator to display AC + CB.

Use XY Attribute point to display C's x-coordinate and the length AC + CB. Create a locus.

**Autograph files:**
**2. Heron - start.agg** and 2**. Heron.agg**

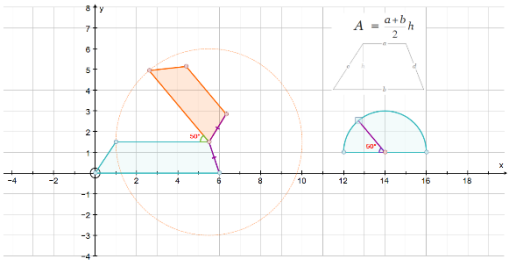
## 3. Battleships with vectors

*After what time will they be closest to each other?*
Ship A: at (–3,6), velocity [1, –1] -> displacement [t, –t]
Ship B: at (–1,0), velocity [1.5, 2] -> displacement [1.5t, 2t]
Reduce B to rest by adding a negative vector.

Add that vector to A's vector -> A's motion relative to B
Select A's vector and the point B: draw perpendicular
Use constant controller to vary 't'.

**Autograph files:**
**3. Battleships - start.agg** and **3. Battleships.agg**

## 4. Area of trapezium

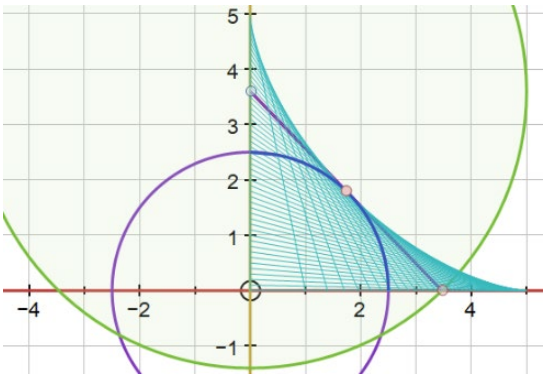


Enter the trapezium's 4 vertices as points.
Create the mid-points of the sloping sides.
Create a shaded area for the bottom half.
Group the 4 vertices of the top half to a shape.

Create an angle controller based on a semi-circle, and use "Manage constants" to associate ϕ.
Make a point as centre of rotation, select it and the shape to rotate clockwise through ϕ.
Move the centre onto the bottom right vertex. Control the rotation from the semi-circle.

**Autograph files:** **4. trapezium – start.agg** and **4. trapezium.agg**

## 5. Falling ladder – locus

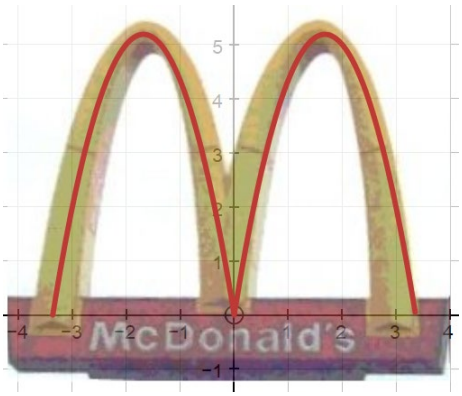


Draw x = 0, place point A at (0,4), and circle radius 5
Draw y = 0, and find its intersection with the circle, B
Hide the circle. Draw the ladder: segment AB.
Use 'Edit Draw Options' to thicken up the ladder.
Move 'A' up and down. Select 'A' and the ladder:
Create -> locus (from y = 0 to y = 5, step 0.2)

*Further research:*
Locus of the mid-point is $x = 2.5\cos\theta$, $y = 2.5\sin\theta$
Envelope of the ladder is $x = 5\cos^3\theta$, $y = 5\sin^3\theta$

**Autograph files:** **5. Falling ladder - start.agg** and **5. Falling ladder.agg**
Also: **5a. circle-locus.agg**

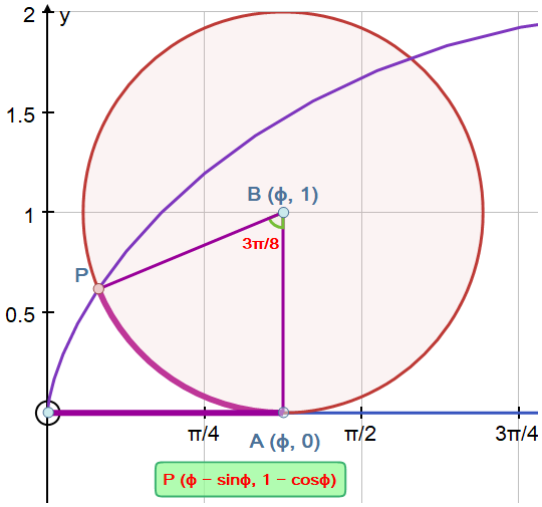
## 6. Fitting McDonald's




Paste in the image (untick 'scale image with axes')
Place the origin at the base of the central section
Enter y = ax(b – x) and find 'a' and 'b' to fit.
Visit Start-up Options
set 'Manual': x-start: 0, x-finish: b
To get the left 'branch', replace 'x' with |x|

**Autograph files:**
**6. McDonalds - start.agg** and **6. McDonalds.agg**

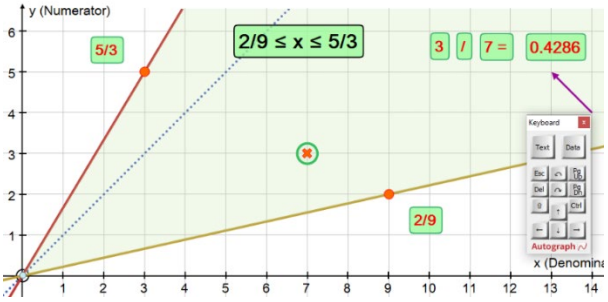
## 7. Cycloid construction




Set up x-axis with π-scales, equal aspect.
Draw y = 0 and point 'A' on this axis
Use "Manage constants" to set ϕ to x-coordinate
Enter point 'B' (ϕ, 1). Circle centre 'B', radius 1
Select 'B' then 'A': Create angle (hence point 'P')
enter ϕ, clockwise, allow reflex
Select 'P', 'B', 'A': Circle -> Arc (with centre)

**Autograph files: 7. Cycloid - start.agg** and **7. cycloid.**

## 8. Fractions




To explore fractions between 2/9 and 5/3
Set up axes as illustrated
Set x- and y-snaps to 1
Draw y = (5/3)x and y = (2/9)x and y = x
Place the points (3,5) and (9,2)

Place a random point 'P' in the middle, eg (7,3)
and use the calculator to display 3, 7 and the result, and the text box to display '/'
To shade the area, select upper then lower lines then Create -> Area from 0 to 20

Explore by moving 'P' about

**Autograph files:**
**8. Fractions - start.agg** and **8. fractions.agg**

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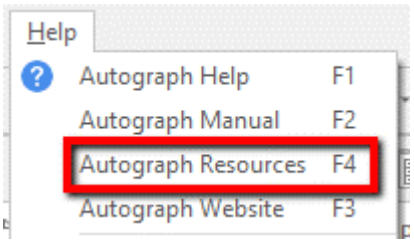


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